Unit 3 volume notes

Introduction

Area and volume questions often involve "problem solving" where one has a wordy question that may need answering in several stages. The most reliable method for such questions is to (i) **write an equation**, then (ii) **solve it**. To help you, many examination papers start with a page of formulae.

Volume

Volume of a prism = (base area) × height

(A prism is anything with a constant cross-section, like a stick of Blackpool rock).

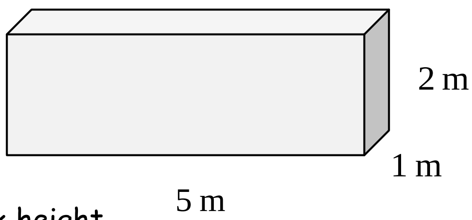
Prisms can be:

- a cuboid (rectangular cross-section)
- a cylinder (circular cross-section)

but also triangular prisms, hexagonal prisms etc

Examples

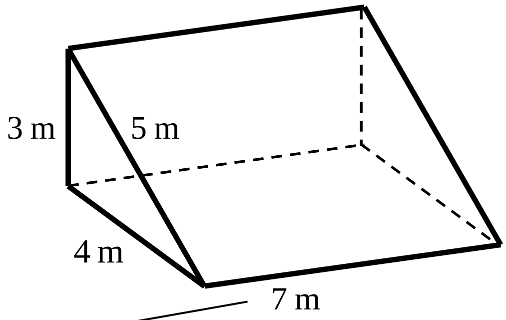
Volume of a cuboid.



Volume = width × depth × height

= 5 × 1 × 2 = 10 m^3.

Volume of a triangular prism.



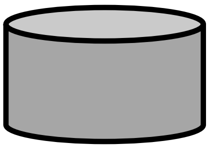
Cross-section area = $\frac{wh}{2} = \frac{3 \times 4}{2} = 6 \text{ m}^2$

Volume = A×L= 6 m^2 × 7 m = 42 m^3.

Cylinders

(i) Diameter D=6m, height h=5 m

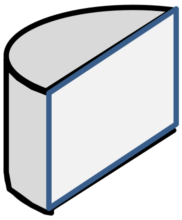
Volume $\pi r^2 h = \pi \times 3^2 \times 5 = 141.4 \text{ m}^3$



$r = 3 \text{ m}$

(ii) <u>Half of this cylinder, sliced vertically</u>

Volume $\frac{1}{2}\pi r^2 h = \pi \times 3^2 \times 5 \div 2 = 70.7 \text{ m}^3$ (half the whole-cylinder value)



(iii) Volume of a thick-walled pipe

A steel pipe has an outer diameter of 200mm, "bore" (the internal diameter) = 170 mm and length 3 m. Find the volume **of steel** used in making the pipe.

External radius = $\frac{200}{2} = 100 \text{ mm} = 0.1 \text{ m}$

Internal radius = $\frac{170}{2} = 85 \text{ mm} = 0.085 \text{ m}$

Volume = (cross-section area)×length

$=\left(\pi\times0.1^2-\pi\times0.085^2\right)\times3$

= 0.0262 m^3.

Your calculation must use the same units throughout (all metres, or all mm)

Problems where a length must be found

- Decide which of the above circumference, area of volume formulae you need
    - *If unsure, I suggest you sketch the shape as you read the question, marking the dimensions on your sketch*
- Write an equation in the form "formula = value"
- Solve the equation

Examples:

(a) A cylinder has radius 5 m and volume 200 m^3. Find the length.

$\pi r^2 L = 200$ **(equation)**

$L=\dfrac{200}{\pi\times5^2}=2.546\text{ m}$

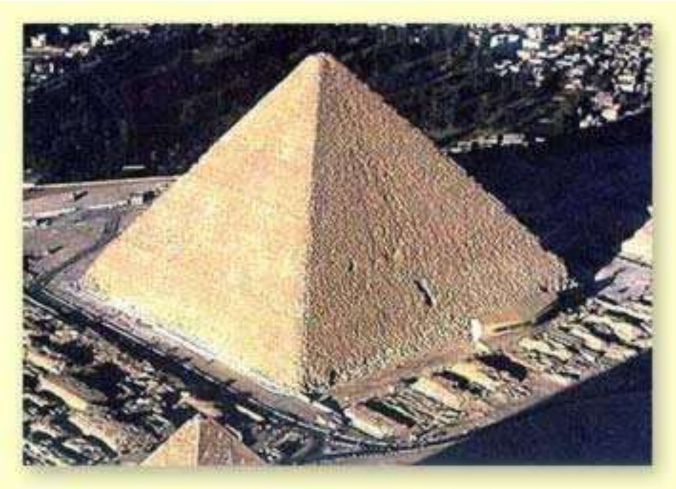
(b) A cylinder has volume 20 m^3 and length 3 m. Find the radius.

$\pi r^2 L = 20$ **(equation)**

$r^2=\dfrac{20}{\pi L}=\dfrac{20}{3\pi}$ on calculator, then $r=\sqrt{ANS}=1.457\text{ m}$

Pyramids

- Volume of any pyramid = $\frac{1}{3}(\text{base area})\times(\text{vertical height})$
    - e.g. volume of a cone = $\frac{1}{3}\pi r^2 h$ where h is the vertical height.



The Great Pyramid of Khufu has sides of length 230 m and is 147 m high
It contains 2.4 million blocks of stone, density 2400 kg/m^3

(a) What is the mass of each block, if they are all the same?
(b) The world production of concrete is 6x10^9 m^3/year. How many pyramids per year could this produce?

You need to plan your method:

- find the total volume
- find the total mass, using the volume and density
- then find the mass of each block, based on the number of blocks
- finally, use proportion to find how many concrete pyramids could be built

Volume = $\frac{1}{3}(\text{base area})\times\text{height} = \frac{230\times230\times147}{3} = 2592100\ \text{m}^2 = 2.592\times10^6\ \text{m}^3$

Mass = $\text{volume}\times\text{density} = 2.592\times10^6\times2400 = 6.22\times10^9\ \text{kg}$

Use ANS button to continue calculation

Mass of each block = $\frac{\text{total mass}}{\text{number of blocks}} = ANS \div 2.4\times10^6 = 2592\ \text{kg} \approx 2.6\ \text{tonnes}$

Be <u>very careful</u> to use the $\times10^x$ or EXP button to enter a standard form number, not the $10^{\bullet}$ button

Number of pyramids/year = $\dfrac{6\times10^{9}\ \ \text{m}^3/\text{yr}}{2.6\times10^{6}\ \ \text{m}^3/\text{pyramid}} = 2308$

Spheres

- Volume of a sphere = $\dfrac{4}{3}\pi r^3$

<u>Example</u>

The moon is 2000 miles diameter. What is its volume? Give your answer in standard form.

$r = \dfrac{D}{2} = 1000 \text{ miles}$

$V = \dfrac{4}{3}\pi r^3 = 4\pi \times 1000^3 \div 3 = 4.19\times10^9 \text{ cubic miles}$

Tip: The ENG button will display a number as a multiple of $10^3, 10^6, 10^9$ etc. This is not necessarily "standard" form but it makes the conversion easier. Try it! Pressing ENG again reduces the exponent in steps of 3 (giga, mega, kilo etc), pressing ← increases the exponent.