Final revision for Unit 3

1. $2x+13=17$

   $2x=17-13=4$

   $x=4\div 2=2$

2. $2x-11=35$

   $2x=35+11=46$

   $x=46\div 2=23$

3. $8x+7=2x+79$ $-2x$

   $6x+7=79$ -7

   $6x=72$

   $x=72\div 6=12$

4. $\frac{1}{3}x+15=12$

   $\frac{1}{3}x=12-15=-3$

   $x=-3\times 3=-9$

5. $7(2x-5)=49$ $\div 7$

   $2x-5=7$

   $2x=12$

   $x=12\div 2=6$

6. $12(17-x)=5(2x+1)+1$

   $204-12x=10x+5+1$

   $204=22x+6$

   $22x=204-6=198$

   $x=198\div 22=99\div 11=9$

7. $\frac{2x+1}{7}=33-3x$

   $2x+1=231-21x$

   $23x+1=231$, $23x=231-1=230$

   $x=230\div 23=10$

8. $\frac{1}{2}x + \frac{1}{3}x - \frac{1}{5}x - \frac{1}{7} = \frac{3}{4}$

$$\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{5}\right)x = \frac{3}{4} + \frac{1}{7}$$

$$\left(\frac{15+10-6}{30}\right)x = \frac{21+4}{28}$$

$$\frac{19}{30}x = \frac{25}{28}$$

Now $\div \, ^{19}/_{30}$, same as $\times \frac{30}{19}$

$$x = \frac{25}{28} \times \frac{30}{19} = \frac{375}{266}$$

9. $x^2 - 9x + 20 = 0$ Factors of 20

$-1 \quad -20$
$-2 \quad -10$
$-4 \quad -5$ adds to $= -9$

$$(x-4)(x-5) = 0$$

$\therefore x - 4 = 0, \quad x = 4$
or $x - 5 = 0, \quad x = 5$

10. $x^2 + 7x - 60 = 0$ Factors of -60

-1×60
-2×30
-3×20
-4×15
-5×12 ✓

$$(x-5)(x+12) = 0$$

$\therefore x - 5 = 0, \quad x = 5$
or $x + 12 = 0, \quad x = -12$

11. $2x^2 - 3x - 2 = 0$ $\quad ac = -4 = -4 \times 1 \; (-4+1=-3)$

$$\left(2x + \frac{1}{1}\right)\left(1x - \frac{4}{2}\right) = 0,$$
$$= (2x+1)(x-2)$$

$\therefore 2x + 1 = 0, \quad x = -\frac{1}{2}$
OR $x - 2 = 0, \quad x = 2$

12. $6x^2 - 11x - 10 = 0$ $\qquad ac = -60$

-60×1
-30×2
-20×3
$-15 \times 4 \quad -15+4 = -11$ ✓

$\left(3x + \frac{4}{2}\right)\left(2x - \frac{15}{3}\right)$
$= (3x+2)(2x-5)$

or

$\left(6x + \frac{4}{1}\right)\left(1x - \frac{15}{6}\right) = (6x+4)(x-2.5)$ (equivalent!).

Then

$3x+2=0, \quad x = -2/3$

or $\quad 2x-5=0, \quad x = 5/2$

13. $(x+5)(x-5) = 119$ $\qquad$ "difference of two squares"

$x^2 - 25 = 119$

$x^2 = 144$

$x = \sqrt{144} = \pm 12$

14. $3x^2 - 4x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16+60}}{6}$$

$$= \frac{4 \pm \sqrt{76}}{6}$$

$\therefore \; x = \frac{4+\sqrt{76}}{6} = 2.120$ (round from 2.11963....)

or $x = \frac{4-\sqrt{76}}{6} = -0.7863$

15. $x^2 - x = 56$ $\qquad$ Spot the x^2 – it's a quadratic!

$x^2 - x - 56 = 0$ $\qquad -56$

$(x-8)(x+7) = 0$

-56×1
-28×2
-14×4
-8×7 ✓
-7×8

$x-8=0, \; x = 8$

or $\; x+7=0, \; x = -7$

16. $2x - 1 = \frac{21}{x}$ $\quad \times x$ to lose the fraction

$2x^2 - x = 21$ $\quad$ Quadratic!

$2x^2 - x - 21 = 0$ $\quad ac = -42 = -7 \times 6$

$\left(2x - \frac{7}{1}\right)\left(1x + \frac{6}{2}\right) = (2x-7)(x+3) = 0$

$\therefore x = 7/2$ or -3

17. $\frac{4}{x} = \frac{2x-3}{x+1}$

$(\times x)$: $4 = \frac{x(2x-3)}{x+1}$

$\times (x-1)$: $4(x+1) = x(2x-3)$

$4x + 4 = 2x^2 - 3x$

$\therefore 2x^2 - 7x - 4 = 0$

$ac = -8 = -8 \times 1$

$\left(2x + \frac{1}{1}\right)\left(1x - \frac{8}{2}\right) = (2x+1)(x-4) = 0$

$\therefore x = -1/2,\ x = 4$

18. $\frac{x}{4} = \frac{2x+3}{x+1}$

Cross-multiply: $x(x+1) = 4(2x-3)$

$x^2 + x = 8x - 12$

$x^2 - 7x + 12 = 0$

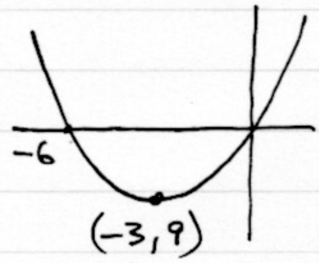
$(x-3)(x-4) = 0$

$x = 3,\ x = 4.$

19. $x^2+6x = (x+3)^2 - 9$

halve

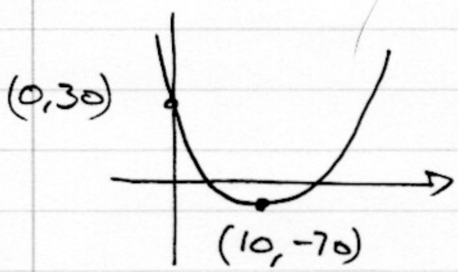
square & subtract



Minimum $y=-9$
because $(x+3)^2$ can be 0 but not negative.
To make $x+3=0$, need $x=-3$

20. $x^2-20x+30 = (x-10)^2 - 100 + 30$

$= (x-10)^2 - 70$

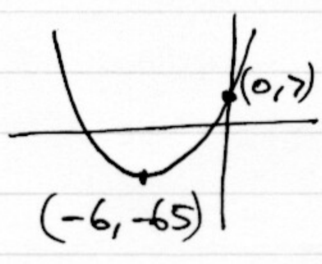


21. $2x^2+24x+7 = 2\left[x^2+12x\right]+7$

$=2\left[(x+6)^2-36\right]+7$

$= 2(x+6)^2 - 72 + 7$

$= 2(x+6)^2 - 65$



22. $y = 2x+1$

$\underline{y = 7-x} \quad -$

$0 = 2x-(-x)+1-7$

$= 3x-6$

$\therefore\ 3x=6,\ x=2$

$y = 2x+1 = 5$.

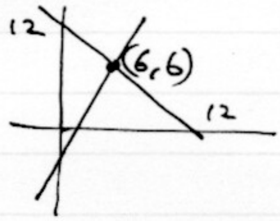
23. $x+y=12$ "$+y$ and $-y$", add to eliminate y.

$2x-y=6$ +

$3x=18$ $\quad x=6$

$6+y=12, \quad y=6$

Lines intersect at $(6,6)$.



24. $5x+3y=6 \xrightarrow{\times 3} 15x+9y=18$

$3x+8y=47 \xrightarrow{\times 5} \cancel{15x}+40y=235$

$\cancel{15x}+9y=18$ —

$31y=217$

OR

$y=217\div31=7$

$\therefore 5x+21=6$

$5x=6-21=-15$

$x=-3$

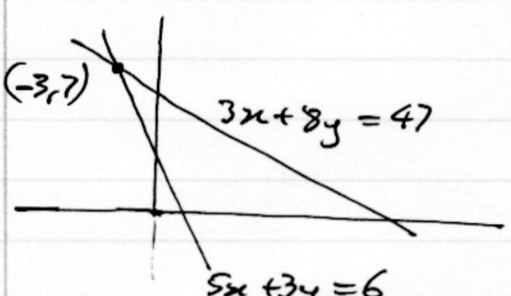
(×8) $40x+\cancel{24y}=48$

(×3) $9x+\cancel{24y}=141$ —

$31x=-93$

$x=-93\div31=-3$

$\therefore -15+3y=6, \quad 3y=21, \quad y=7$



25. $23x=189, \quad x=189\div23=8.21739=8.217$ to 4 s.f.

26. $0.19y=1.75, \quad y=1.75\div0.19=9.210526=9.211$ (4 s.f.)

27. $5\pi x=30, \quad x=\frac{30}{5\pi}=1.909859=1.910$ to 4 s.f.

n.b. NOT $30\div5\pi$

28. $(2.39+\pi)x = 400.57, \quad x = \dfrac{400.57}{2.39+\pi} = 72.4149$

$= 72.41$ to 4 s.f.

29. $(\pi-2)x = \pi+8, \quad x = \dfrac{\pi+8}{\pi-2} = 9.75969$

$= 9.760$ to 4 s.f.

30. $1.6\times10^{-19} N = 180, \quad N = 180 \div 1.6\times10^{-19}$

$= 1.125\times10^{21}$

31. $17x - 91 = 204$

$17x = 204 + 91 = 295$

$x = \text{ANS} \div 17 = 17.3529$

$= 17.35$ (4 s.f).

32. $1837y + 248 = 692$

$1837y = 692 - 248 = 444$

$y = \text{ANS} \div 1837 = 0.241698\ldots$

$= 0.2417$ to 4 s.f.

33. $17(x+2) = 111$

$x + 2 = 111 \div 17$

$x = \text{ANS} - 2 = 4.529411$

$= 4.529$ (4 s.f).

34. $\dfrac{y+87}{31} = 5.92$

$y + 87 = 5.92\times31$

$y = \text{ANS} - 87 = 96.52$

35 $x = \pm\sqrt{167} = \pm 12.92$

36. $x^2 = 5.43 \times 10^7$ $\quad x = \sqrt{5.43 \times 10^7} = 7368.85$

$= 7369$ to 4 sig. figs

37. $216x^2 = 8.31 \times 10^{-5}$

$x^2 = 8.31 \times 10^{-5} \div 216$

$x = \sqrt{ANS} = 6.202598 \times 10^{-4}$

$= 0.0006203$ to 4 s.f.

38. $9.8\pi r^2 = 22.7$, $\quad r^2 = \dfrac{22.7}{9.8\pi}$

$r = \sqrt{ANS} = 0.858667$

$= 0.8587$ to 4 s.f.

~~39~~ 40 $r^3 = 583.8$, $\quad r = \sqrt[3]{583.8} = 8.357724$

$= 8.358$ to 4 sig. figs

39. $12.8(7.37 + r^2) = 113.4$

~~40.~~ $7.37 + r^2 = \dfrac{113.4}{12.8}$, $\quad r^2 = \dfrac{113.4}{12.8} - 7.37$

$r = \sqrt{ANS} = 1.2203995$

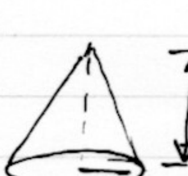
$= 1.220$ to 4 s.f

41. $\frac{4}{3}\pi r^3 = 200$, $\quad \pi r^3 = 200 \times \frac{3}{4} = 150$

$r^3 = \dfrac{150}{\pi}$, $\quad r = \sqrt[3]{ANS} = 3.62783$

$= 3.628$ to 4 s.f.

42.



$h = 9\text{cm}$

$r = \frac{10}{2} = 5\text{cm}$

$V = \frac{1}{3}$ (base area) $\times$ height

$= \frac{1}{3}\pi r^2 h$

$= \dfrac{\pi \times 25 \times 9}{3} = 235.619$

$= 235.6\ \text{cm}^3$ (4 s.f.).

43.

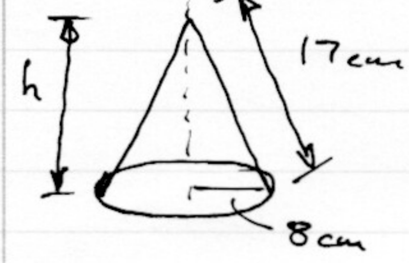
9, 5, $\sqrt{81+25} = \sqrt{106}$

Curved area $\pi r l = \pi \times 5 \times \sqrt{106}$

Circular base $\pi r^2 = 25\pi$

Total $25\pi + 5\pi\sqrt{106} = 240.263 \approx 240.3\ \text{cm}^2$

44.



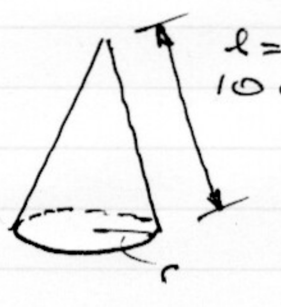
$h^2+8^2=17^2$

$h^2=17^2-8^2=225 \therefore h=15$ cm

$V=\frac{1}{3}(\pi \times 8^2)\times 15 = 1005.3$

≈ 1005 cm^3 to 4 s.f.

45.



Surface area $\pi r^2+\pi r l$

$=\pi(r^2+10r)=600$

Quadratic equation!

$\pi r^2+10\pi r-600=0$

$$r=\frac{-10\pi \pm \sqrt{100\pi^2-4\times\pi(-600)}}{2\pi}$$

$$=-5 \pm \frac{\sqrt{100\pi^2+2400\pi}}{2\pi}$$

Use the + (other root is negative, not useful)

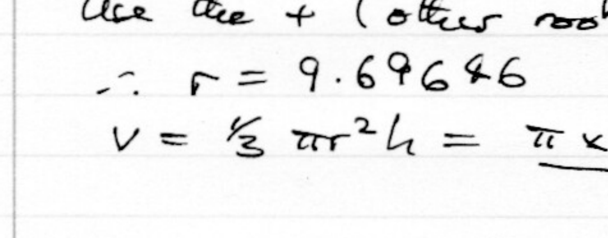
$\therefore r=9.69646$

$$V=\frac{1}{3}\pi r^2 h = \frac{\pi \times \text{ANS}^2\sqrt{10^2-\text{ANS}^2}}{3}$$

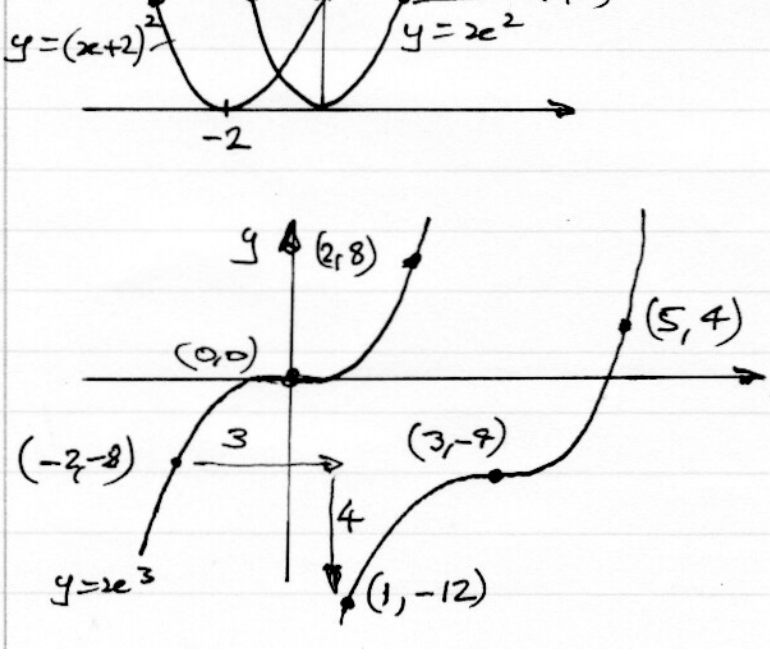
$=240.745 \approx 240.7$ cm^3 to 4 s.f.

or 241 cm^3 to 3 s.f.

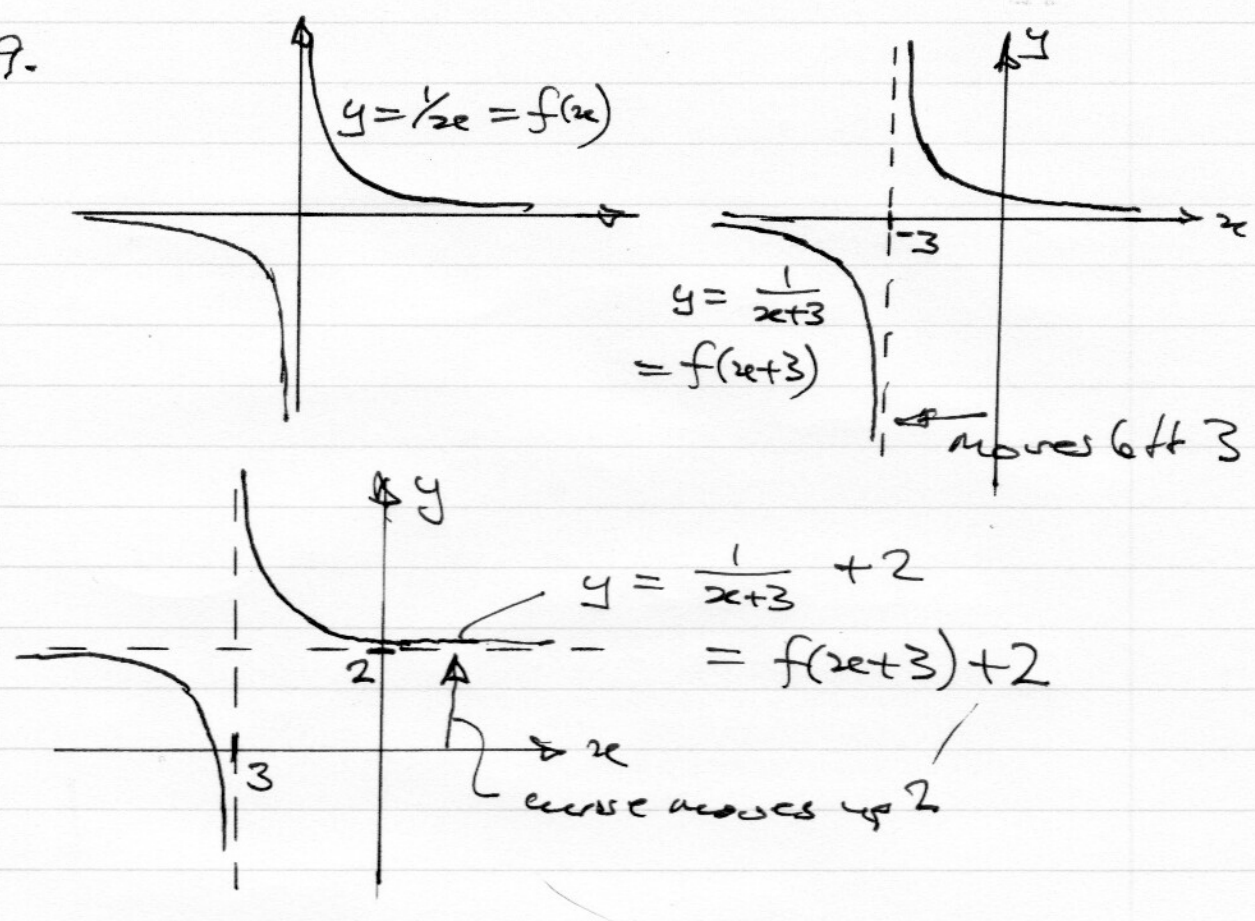
46.



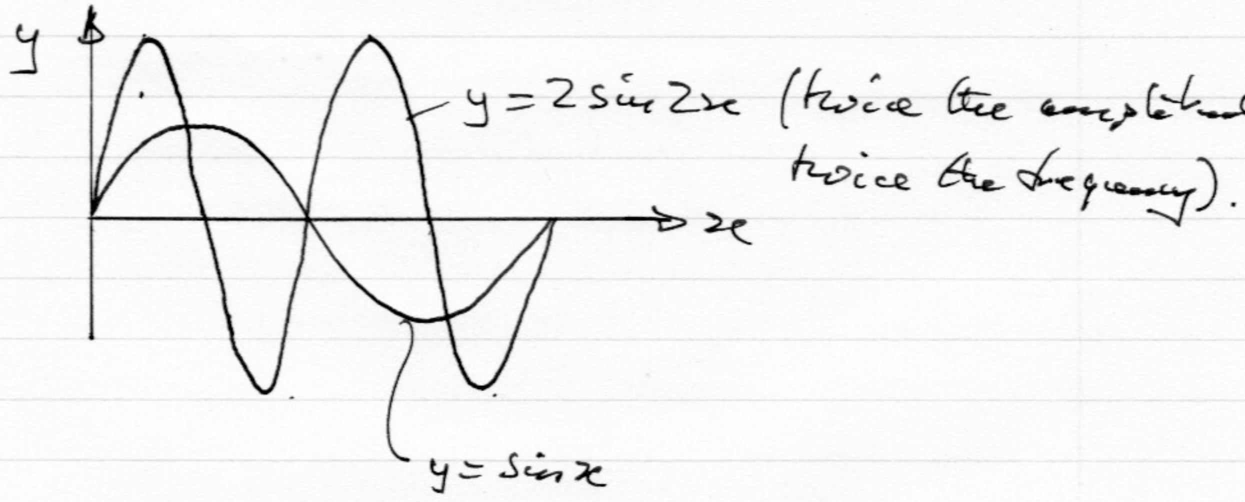
47.



49.



48.



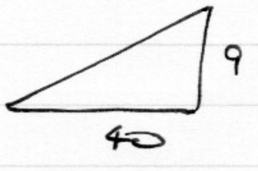
50. Volume $\propto$ mass (constant density)
6 tonnes = 6000 kg.

Volume of adult $= \frac{6000}{60} = 100 \times$ vol of baby.

Length of adult $= \sqrt[3]{100} \times$ length of baby

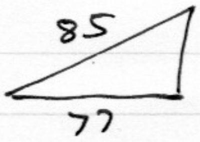
Area of ears $\propto$ length^2 so $\left(\sqrt[3]{100}\right)^2 \times 2000 \approx 43100\ \text{cm}^2$

51.



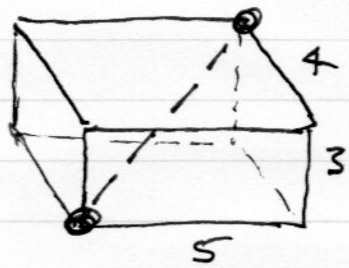
$$\sqrt{40^2+9^2} = 41$$

52.



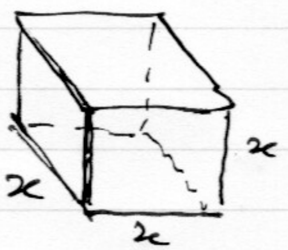
$$\sqrt{85^2-77^2} = 36$$

53.



$$\sqrt{3^2+4^2+5^2} = \sqrt{50}$$
$$= 7.071 \text{ cm}$$

54



$$\sqrt{x^2+x^2+x^2} = 6$$

$$3x^2 = 6^2 = 36$$
$$x^2 = 12$$
$$x = \sqrt{12} = 3.464 \text{ m}$$

55. Price $\times 1.43 = £12.50$

Price was $\dfrac{£12.50}{1.43} = £8.74$

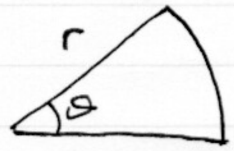
56. $I = \left(\dfrac{350}{27}\right)^{-1} \times 35 \text{ mA} = 2.7 \text{ mA}$

since inversely proportional

57. $S = D/T$. Max speed $\dfrac{100.5\text{m}}{19.5\text{sec}} = 5.154 \text{ m/s}$,

minimum speed $\dfrac{99.5\text{m}}{20.5\text{s}} = 4.854 \text{ m/s}$

58.



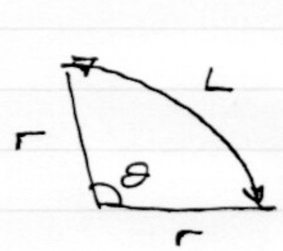
$r = 5\text{cm}$

$\text{area} = \left(\frac{\theta}{360}\right) \times \pi r^2 = 15\text{cm}^2$

$\therefore \frac{\theta}{360} = \frac{15}{25\pi}$

$\theta = \frac{15}{25\pi} \times 360 = 68.75^\circ$

59.



Perimeter $= 2r + L =$ "P".
and $L = P/2$

$2r = P - P/2 = \frac{1}{2}P, \; P = 4r,$
$L = 2r.$

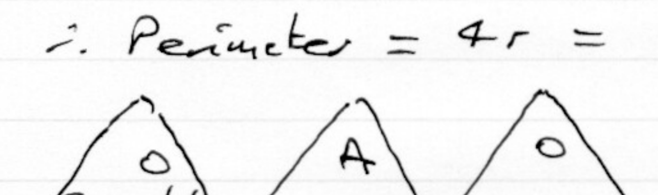
Angle $\theta = \frac{2r}{2\pi r} \times 360^\circ,$

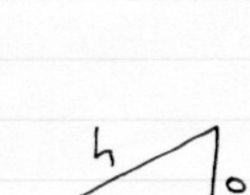
$\frac{\theta}{360} = \frac{2r}{2\pi r} = \frac{1}{\pi}$ = fraction of circle.

$\therefore 100\text{ cm}^2 = \frac{1}{\pi}(\pi r^2) = r^2, \quad \underline{r = 10\text{ cm}}$

$\therefore$ Perimeter $= 4r = 40$ cm

60





$o = h \times \sin(x)$ $\qquad x = \sin^{-1}\left(\frac{o}{h}\right)$

$o = a \times \tan(x)$

$h = \frac{o}{\sin(x)}$ $\qquad x = \cos^{-1}\left(\frac{a}{h}\right)$

$h = \frac{a}{\cos(x)}$ $\qquad x = \tan^{-1}\left(\frac{o}{a}\right)$

$a = h \times \cos(x)$

$a = \frac{o}{\tan(x)}$