"How to" sheet: Curve Transformations

Try to understand the logic behind transformations rather than just remembering a set of rules.

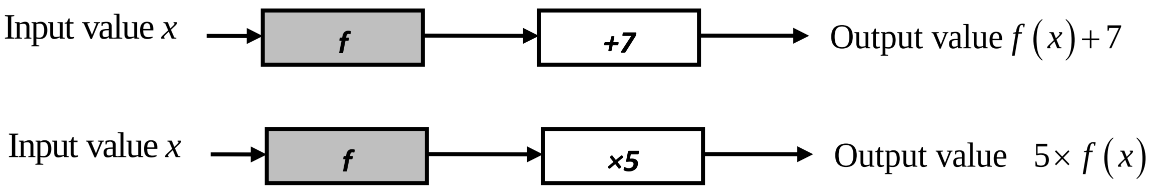
- All your curve transformations either translate or stretch the curve
- A transformation involving multiplication must be a stretch (zero values are still zero, don't move!)
- A transformation involving adding or subtracting must therefore be a translation.

Imagine that you know one pair of coordinates on the curve $y = f(x)$, perhaps (2,7).

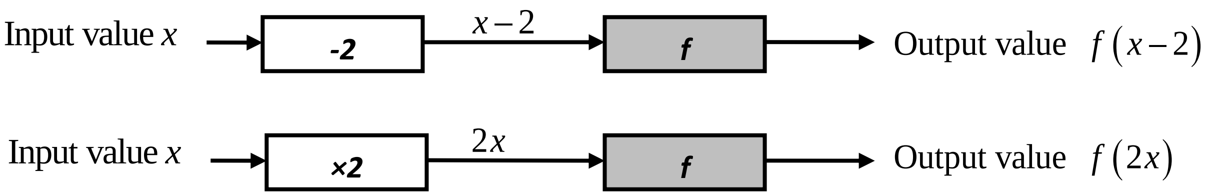
This defines $f(2) = 7$.

Now you are asked to do a transformation. Think: does the transformation change the ***input to f*** or the <u>output from</u> *f*?

- Transformations that **first** find $f(x)$, **then** add a constant or multiply by a constant are giving you a new *y*-value at the same *x* as before (translation or stretch in the *y*-direction, points moving along a vertical, **constant *x* line**).



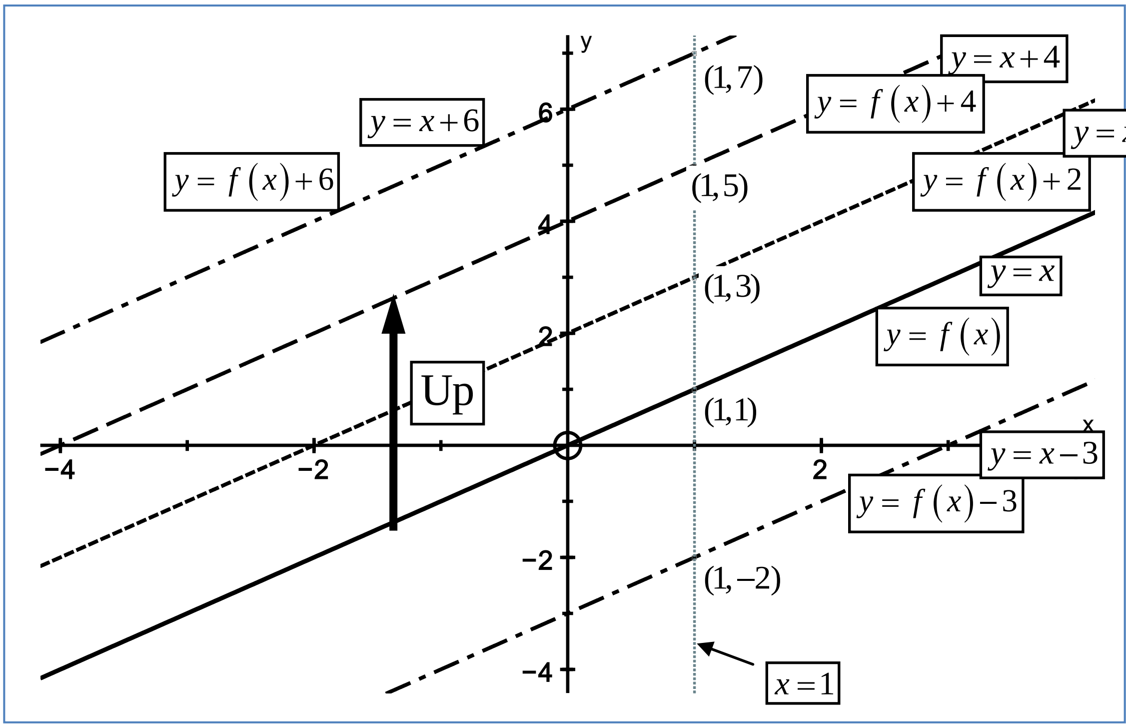
- Transformations that **first** modify the x-value and **then** pass this new number into the function need a different x-value to give you the same y-value as before (translation or stretch in the x-direction, points move along a horizontal, **constant y line**)



	Translation	Stretch
y-direction	$f(x)+a$	$kf(x)$
x-direction	$f(x+a)$	$f(kx)$

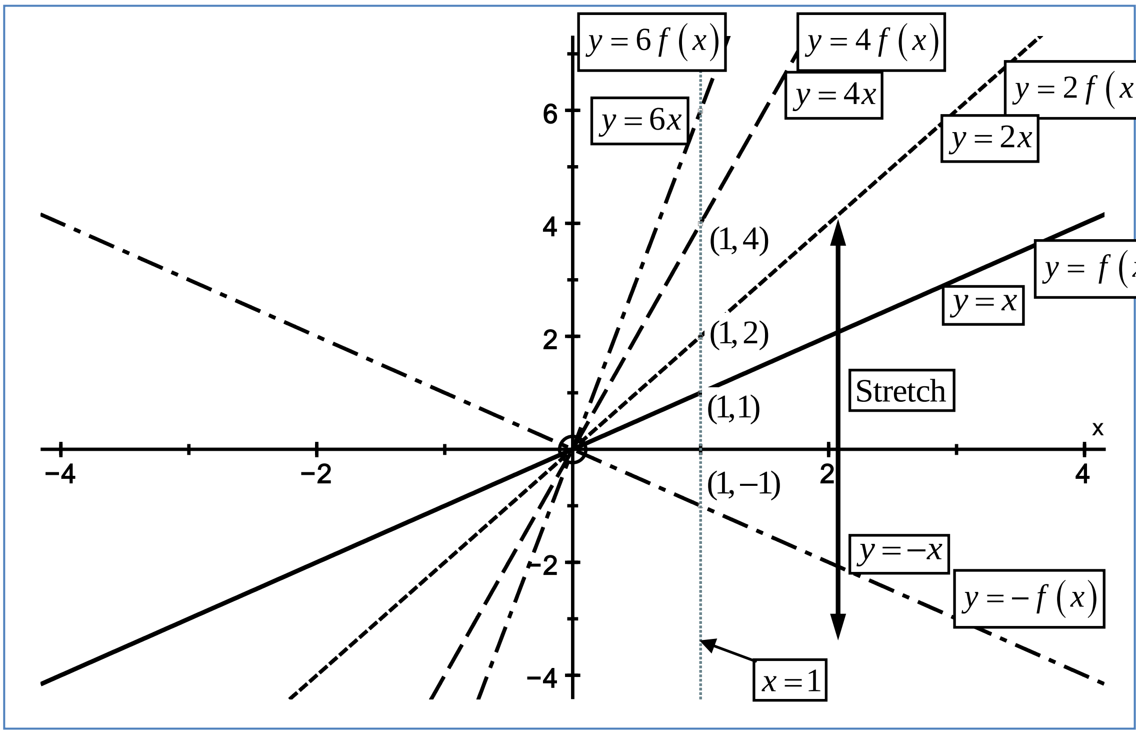
Translation in the y-direction (along a line of constant x)

Defining $f(x) = x$ (a very simple function!) and a point on it with known coordinates, (1, 1). We get a family of lines that are shifted up or down from the initial line. For each line, I will show it both as a transformation of f(x) and as a line equation (just to help you remember it).



Stretch in the y-direction (multiplying all the original y-values by a constant)

Plotting $y = kf(x)$ we get a family of lines with y-values that are multiples of the original y-value. All coordinates are stretched in the y-direction (away from the x-axis) by a factor *k*.

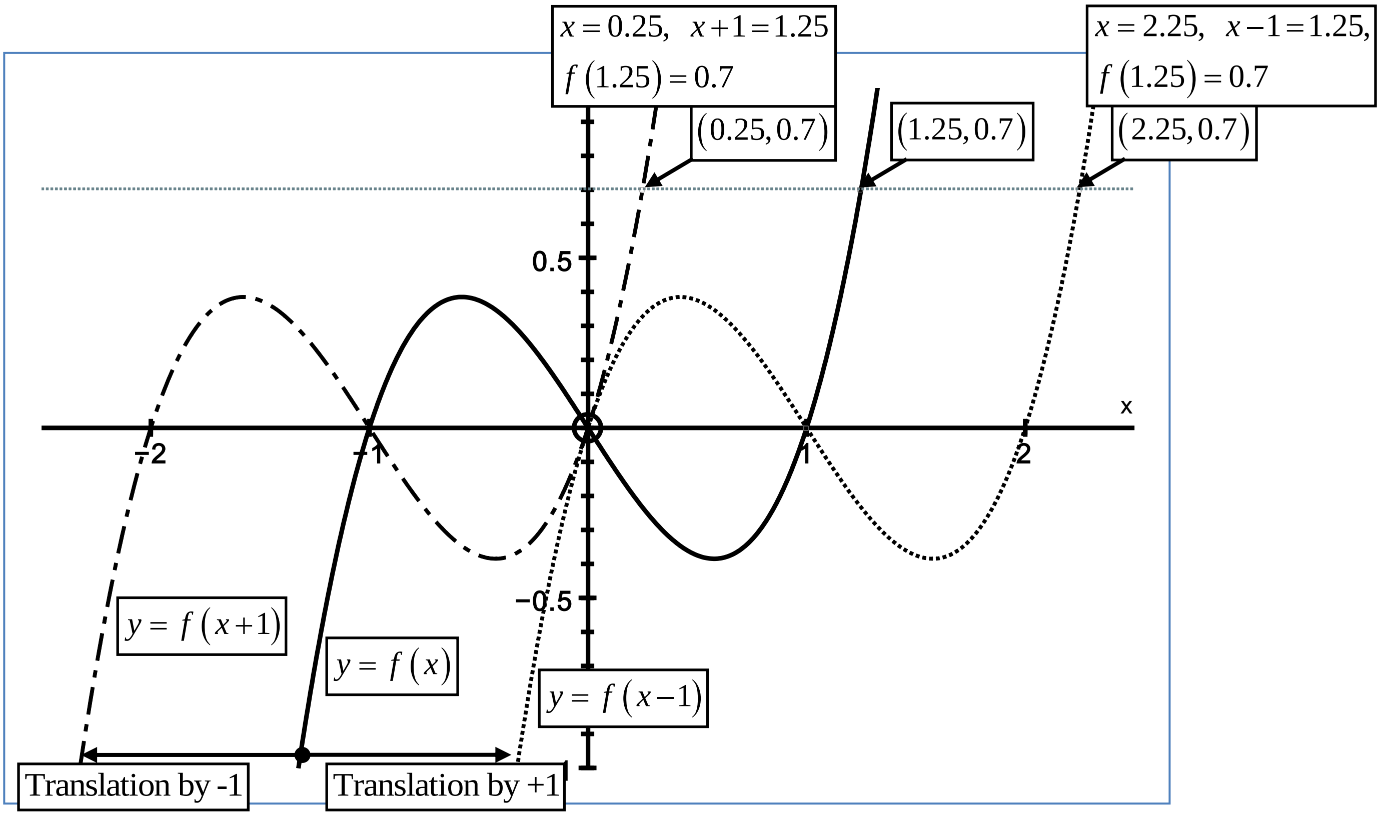


Translation in the x-direction

$y = f(x+a)$ is the curve $y = f(x)$ translated by $-a$ in the $+x$ direction

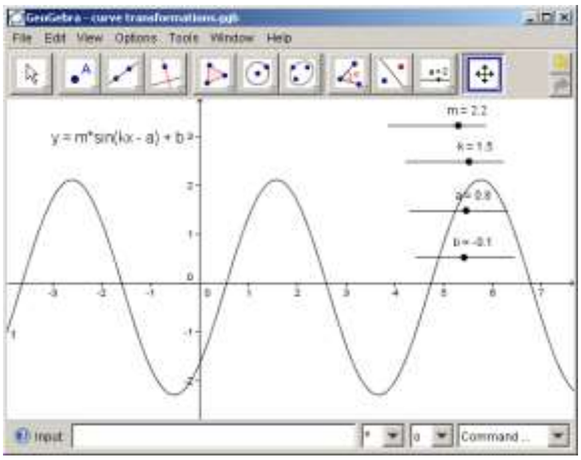
E.g. $y = f(x-1)$, a point moves horizontally $-a = -(-1) = +1$ to the right along a line of constant y. To make this y-value, we must start with a **bigger x-value** than before (because it then has **1 taken off it**).

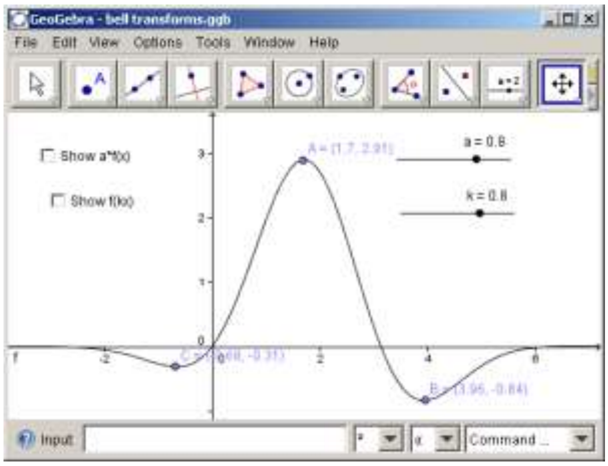
I will take the cubic $f(x) \equiv x^3 - x$ as an example.



Geogebra applets to show you the curves moving
(www.rwmoss.co.uk/Geogebra/curve_transformations_kx+a.ggb, www.rwmoss.co.uk/Geogebra/bell_transforms.ggb)

Geogebra is free from www.geogebra.org.





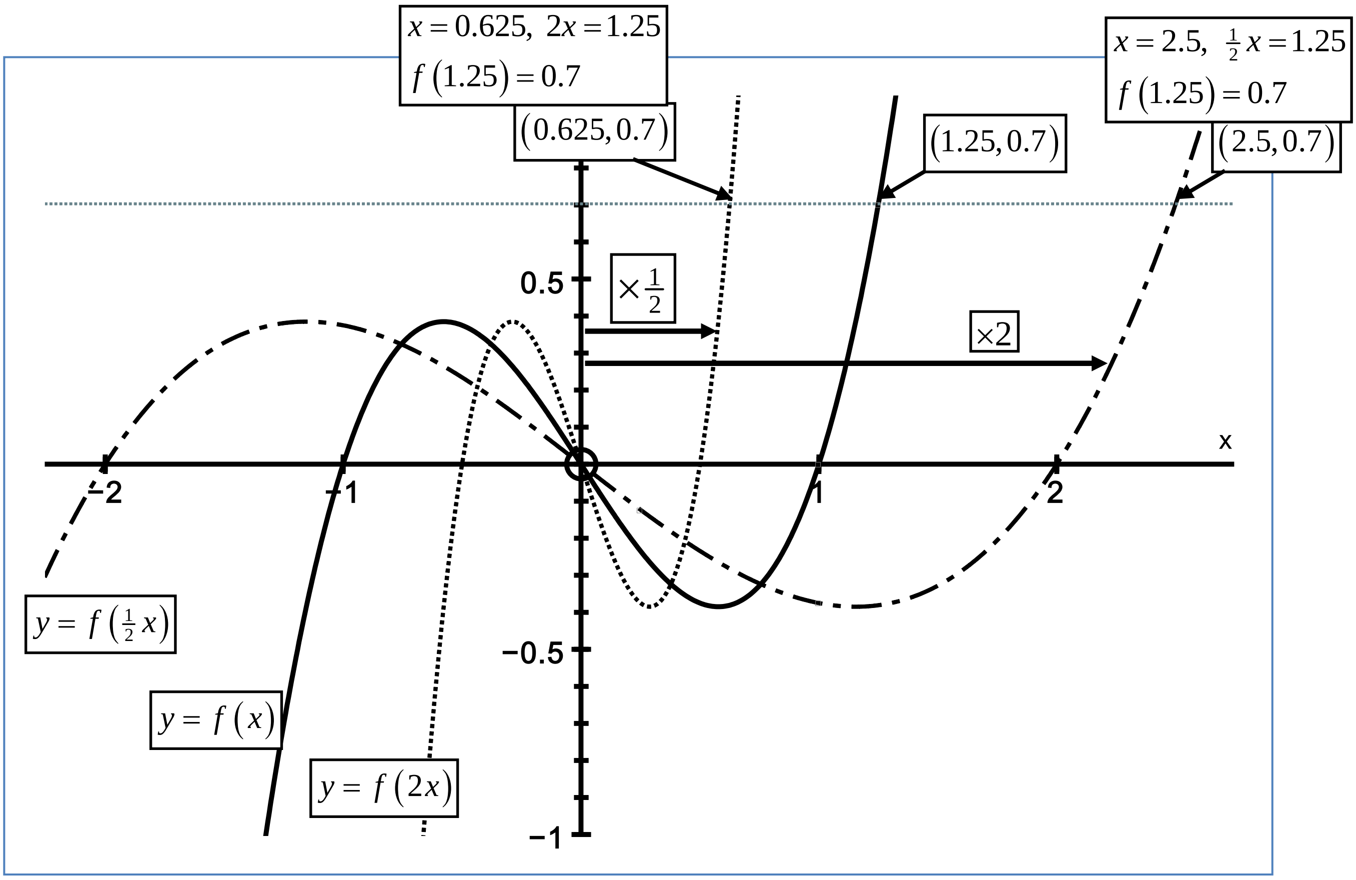
Stretch in the x-direction

$y = f(kx)$ is the curve $y = f(x)$ stretched by a factor $\frac{1}{k}$ in the $+x$ direction.

The frequency is increased by the factor *k* (meaning *k* times as many wiggles in a given x-distance)

E.g. $y = f(2x)$, a point moves **horizontally along a line of constant y.** To make this y-value, we must start with an x-value that is **half of the original** (because it is then doubled).

I will take the cubic $f(x) \equiv x^3 - x$ as an example.



Always check that your x-coordinate, passed through the line definition, gives you the y-value you have chosen (e.g. the $x = 0.625,\ 2x = 1.25,\ f(1.25) = 0.7$ boxes above).