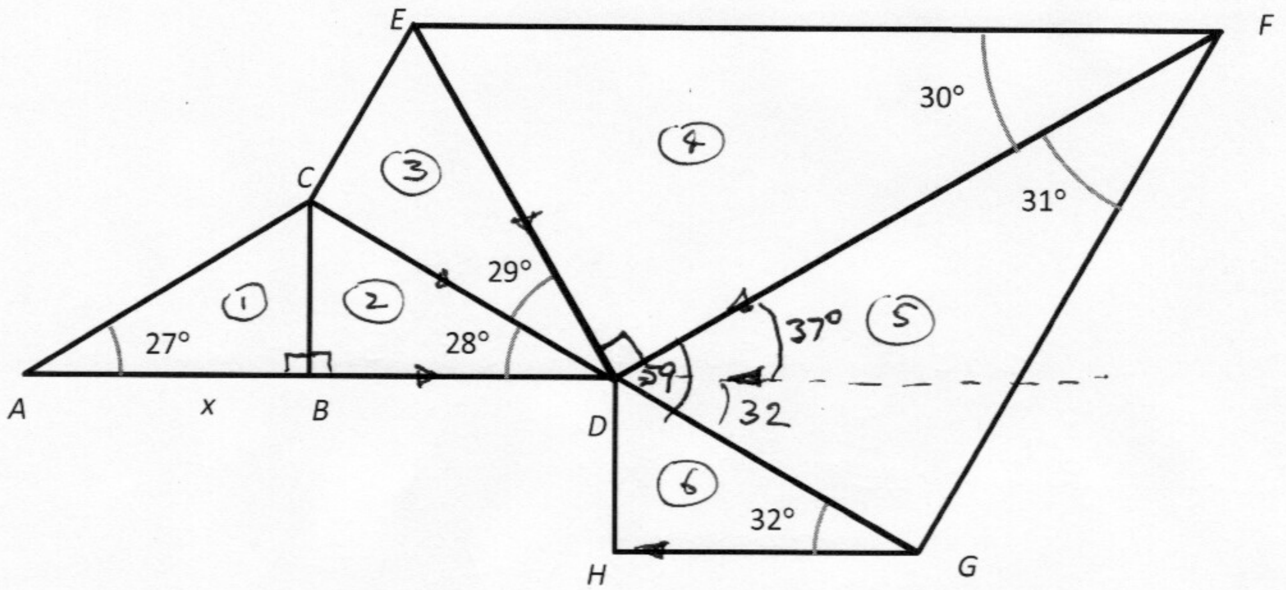
Unit 3 horribly tough revision questions

1. [NOT DRAWN TO SCALE, MOST LINES ARE NOT PARALLEL]

Six **right-angled** triangles are arranged as shown.



(a) The length AB = *x*. Fill in the table, showing the formula defining each length in terms of a previous length, then showing this length as a multiple (to 6 d.p.) of length x. The first one is done for you:

AB = 1x

$BC = x\tan(27) = 0.509525x$	
$BD = \frac{BC}{\tan 28}$	$= 0.958278$ x
$CD = \frac{BC}{\sin 28}$	$= 1.085317$ x
$CE = CD \tan 29$	$= 0.601601$ x
$DE = \frac{CD}{\cos 29}$	$= 1.240902$ x
$DF = \frac{DE}{\tan 30}$	$= 2.149305$ x
$GF = DF \cos 31$	$= 1.842314$ x
$DG = DF \sin 31$	$= 1.106974$ x
$DH = DG \sin 32$	$= 0.586607$ x
$GH = DG \cos 32$	$= 0.938767$ x

1
2
3
4
5
6

multiply width x height coefficients, then add.

(b) Total area = $6.907877\left(\frac{1}{2}x^2\right) = 3.4539\ x^2$

(c) The total area of the six triangles is π^2 square miles. Express x as a multiple of π (to 4 significant figures).

$$3.4539\,x^2 = \pi^2$$

$$\sqrt{3.4539}\;x = \pi$$

$$x = \frac{1}{\sqrt{3.4539}}\pi = 0.5381\pi \text{ miles}$$

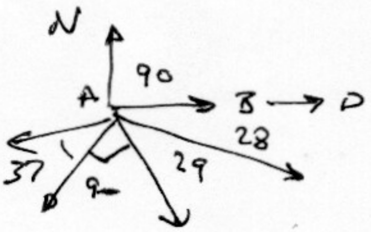
(d) This area is a paddy field (rice farm). The triangles are covered with water. If not replenished, the water level would fall by 1 cm per day through evaporation. What flow rate (litres/second, continuously) is needed to replace the lost water?

1 mile = 1609 m, volume = $1609^2\pi^2 \times 0.01$

$= 252662\ m^2$

Flow rate = $\frac{252662}{24 \times 3600} = 2.92\ m^3/sec = 2920$ litres/sec

(e) Point B is to the East of A. What is the bearing of H from G?



$90 + 28 + 29 + 90 + 37 = 274^\circ$

<u>Note</u>: parallel lines on main diagram

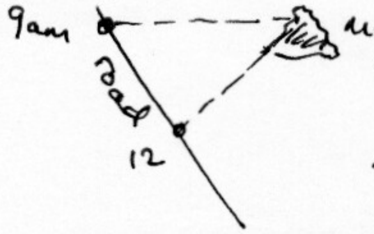
make it easier to visualise the angles.

2. Anatolia is a vast plain with roads running straight for many miles. In the centre (our origin, (0,0)) stands a mountain Erciyes Dagi, visible from far away.

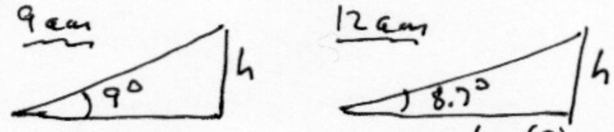
An explorer is riding a camel along a straight horizontal road.

- At 9 am he can see the mountain to his East; the elevation of the summit is 9° and he is a distance x (horizontally) from it.
- At 12 am the summit is North-East from his position and at an elevation of 8.7°

(a) What is his horizontal distance from the summit at 12am, as a multiple of x?



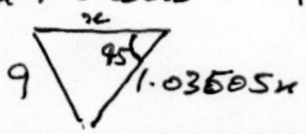
$h = x\tan(9°)$.



$$\frac{x\tan(9)}{\tan(8.7)} = 1.03505x$$

(b) His camel is moving at 3 mph. Find x

3mph × 3 hours = 9 miles



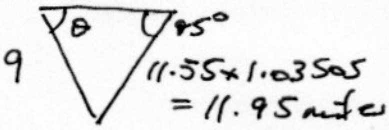
Cosine rule,

$9^2 = x^2 + (1.03505x)^2 - 2.0701x^2\cos 45$

$9^2 = 0.6075x^2$, $x^2 = 133.3$,

$x = 11.55$ miles, $h = 1.83$ miles

(c) On what bearing is he heading as he rides down the road?

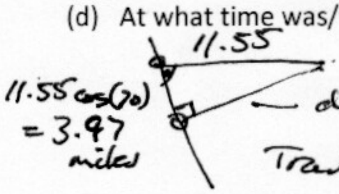


$\frac{\sin\theta}{11.95} = \frac{\sin 45}{9}$, $\theta = 69.88°$ (or use cosine rule).

[Not $180 - 69.88 = 110.12$ ▷ !]

∴ Bearing $= 90 + 70 = 160°$

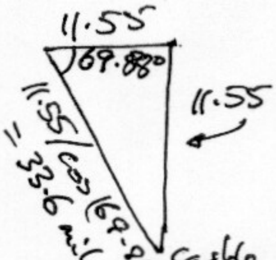
(d) At what time was/will he be closest to the mountain? What is its elevation then?



distance $11.55\sin(69.88) = 10.85$ miles, $\tan^{-1}\left(\frac{1.83}{10.85}\right) = 9.57°$

Travels 4 miles, $1\frac{1}{3}$ hours ∴ 10:20 am

(e) He is heading for a castle that is on the road and due South from the mountain. How far horizontally is the castle from the summit?



$11.55\tan 69.88 = 31.5$ miles

What time does he get there?

33.6 miles/3mph

= 11.2 hours after 9am,

he gets there at 20:12

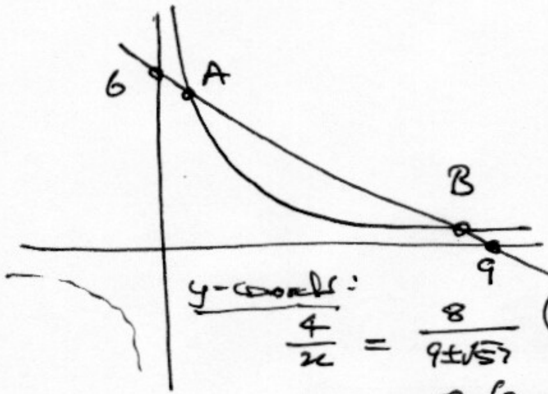
(f) Define an equation for the road (y-axis points Northwards) in the form $y = mx + c$ or $ax + by = c$

Gradient $= -\tan(69.88) = -2.73$, y-intercept -31.5,

$y = -2.73x - 31.5$

OR $\frac{x}{11.55} + \frac{y}{31.5} = -1$

3. The line $2x+3y=18$ cuts the curve $y=\dfrac{4}{x}$ at two points. Sketch the curve and the line (roughly). Find the distance between these points.



$2x + 3(4/x) = 18$

$2x + 12/x = 18$

$2x^2 - 18x + 12 = 0$

$x^2 - 9x + 6 = 0$

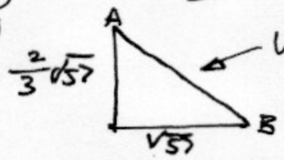
$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{9 \pm \sqrt{81-24}}{2}$

$= \dfrac{9 \pm \sqrt{57}}{2}$

y-coords:

$\dfrac{4}{x} = \dfrac{8}{9 \pm \sqrt{57}} \dfrac{(9 \mp \sqrt{57})}{(9 \mp \sqrt{57})}$

$= \dfrac{8}{24}(9 \mp \sqrt{57})$



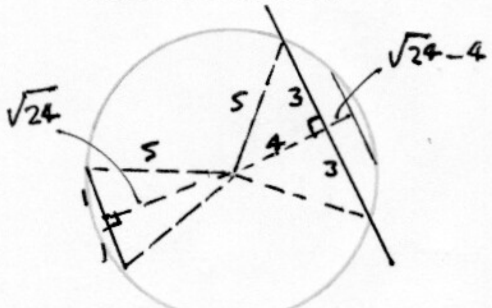
$\sqrt{57}\sqrt{1^2 + (2/3)^2} = \sqrt{57}\sqrt{\frac{13}{9}}$

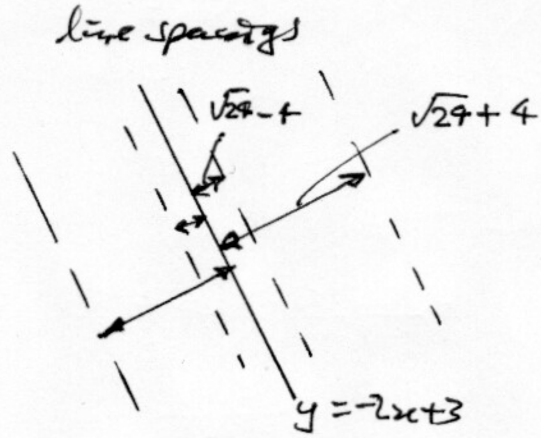
$= \dfrac{\sqrt{741}}{3}$

4. A circle of radius 5 m is cut by a straight line $y=-2x+3$ at points A and B. The distance AB is 6 m.

Another straight line, parallel to the first, cuts the circle at points X and Y such that the distance XY is 2 m.

(a) What line spacings would satisfy this definition? (nb. You do not need to find the centre of the circle or its equation).

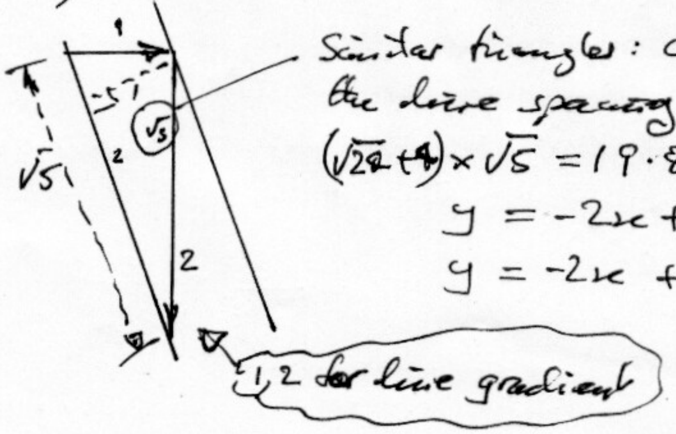




The lines are more than 5 m apart.

(b) Find two possible equations for the second line.

>5m, so $\sqrt{24}+4$ not $\sqrt{24}-4$



Similar triangles: change in y-intercept = $\frac{\sqrt{5}}{1}$ × the line spacing.

$(\sqrt{24}+4) \times \sqrt{5} = 19.899 \approx 19.9$

$y = -2x + (3 + 19.9) = -2x + 22.9$

$y = -2x + (3 - 19.9) = -2x - 16.9$

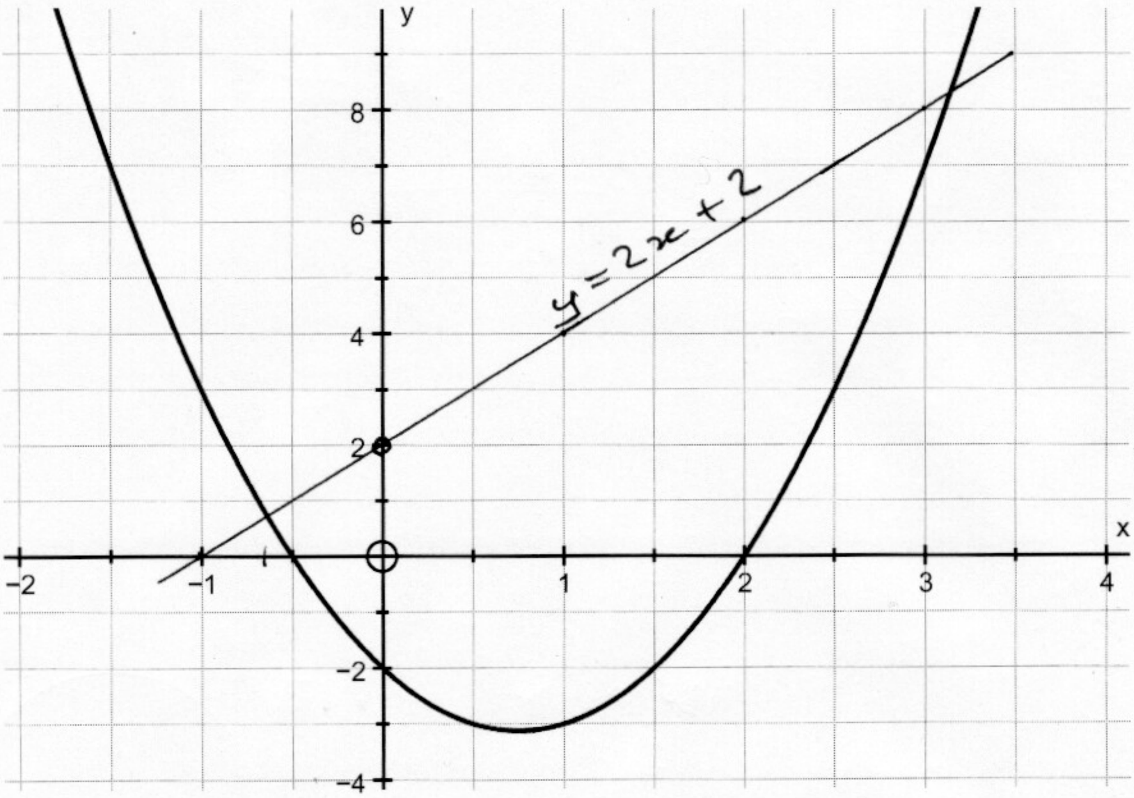
5. The curve $y = 2x^2 - 3x - 2$ is shown below.

(a) Add a straight line to the graph such that the intersection points of the curve and line represent the solutions of $2x^2 - 5x - 4 = 0$, showing the equation of your line.

$2x^2 - 5x - 4 =$
$2x^2 - 3x - 2 - 2x - 2$.

(b) From the graph, estimate the solutions to the equation.

$\therefore y = 2x^2 - 3x - 2 = 2x + 2$.



$x = -0.6, x = 3.1$
(exact $-0.637\ldots$, $3.137\ldots$)

(c) The curve and the line are now translated 2 units to the right.

Write down:

- the new line equation
- the new curve equation

$y = f(x-2)$ is $y = f(x)$ translated $\xrightarrow{2}$

Line $f(x) = 2x + 2$,
and $f(x-2) = 2(x-2) + 2 = 2x - 4 + 2 = 2x - 2$
$\therefore y = 2x - 2$

Curve $f(x) = 2x^2 - 3x - 2$,
$y = f(x-2) = 2(x-2)^2 - 3(x-2) - 2$
[could simplify $\rightarrow$ $y = 2x^2 - 11x + 12$]

6. Solve these equations, giving all solutions in the range $0 \le x < 360°$, to 1 decimal place.

(a) $\sin(x+20) = 0.8$

(b) $\cos(3x) = 0.3$

(c) $\tan(2x+1) = 1$

(a) $x + 20 = \sin^{-1}(0.8) = 53.1°$

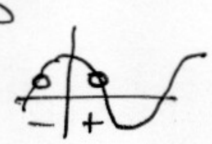
or $180 - 53.1 = 126.9°$ $\Big\}\ \pm n360$

(-20)

$x = 33.1°$, $x = 106.9°$ (+ repeats beyond 360)

(b) $3x = \cos^{-1}(0.3) = \pm 72.54° + n360$

↑ cosine is even



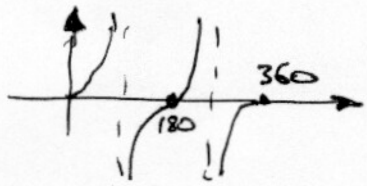
$(\div 3)$

$x = \pm 24.18° + n120$

$\therefore\ x = 24.2°,\ 144.2°,\ 264.2°$

and $\cancel{-24.2°}$, $95.8°$, $215.8°$, $335.8°$

(c) $2x+1 = \tan^{-1}(1) = 45° + n180$



$2x = 44° + n180$

$x = 22° + n90$

$\therefore\ x = 22°,\ 112°,\ 202°,\ 292°$

7(a) $S = D/T$

$$\text{max speed} = \frac{\text{max distance}}{\text{min time}} = \frac{24.5}{7.5}$$

$$= 3.26\dot{6} \text{ mph}$$

$$\text{min speed} = \frac{\text{min distance}}{\text{max time}} = \frac{23.5}{8.5}$$

$$= 2.765 \text{ mph}$$

$$2.76 < \text{Speed} < 3.27$$

(b) $D = S \times T$

max distance $= 3.5 \times 5.5 = 19.25$ miles

min distance $= 2.5 \times 4.5 = 11.25$ miles

(d) Boots are heavy, your leg (like a shorter pendulum) swings faster (more steps/minute) with light shoes!

(c) ~~5.4+~~ $5.5 + 7.5 = 13$ miles max.

$6.5 - 5.5 = 1$ mile min.

(d) $\text{max time} = \frac{D}{S} = \frac{5.5 \text{ miles}}{3.95 \text{ mph}} = 1.39$ hours

$\text{Min time} = \frac{4.5 \text{ mile}}{4.05 \text{ mph}} = 1.11$ hours.