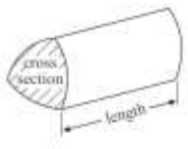
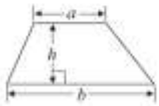
# Unit 3 – surface area.

<u>Formulae provided in exams</u>

**Volume of prism** = area of cross section × length




**Area of trapezium** = $\frac{1}{2}(a+b)h$

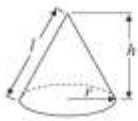



**Volume of sphere** = $\frac{4}{3}\pi r^3$

**Surface area of sphere** = $4\pi r^2$


**Volume of cone** = $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** = $\pi r l$




You must remember the other formulae:

- area of a rectangle
- area of a triangle
- area of a circle
- circumference of a circle

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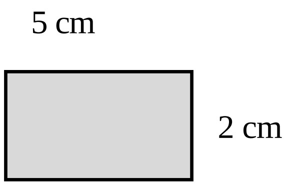
## Areas of common 2-D shapes (revision of unit 2)

**Rectangle**

Word formula: "Area = width × height"

*A = w×h*



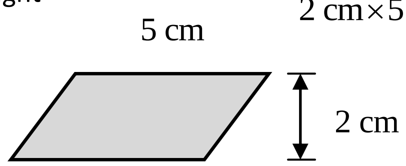

$2\text{ cm}\times 5\text{ cm} = 10\text{ cm}^2$

**Parallelogram**

Word formula: "Area = base width × vertical height"

*A = w×h*

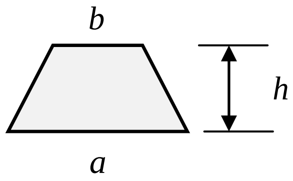
$2\text{ cm}\times 5\text{ cm} = 10\text{ cm}^2$



**Trapezium**

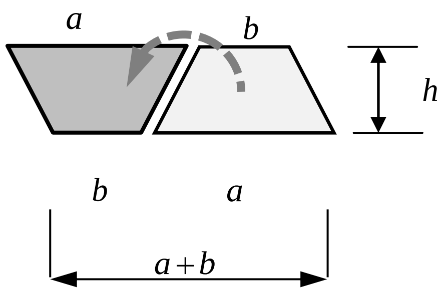
Word formula:  “half the sum of the two parallel sides, times height”

$$A = \left(\frac{a+b}{2}\right)h$$



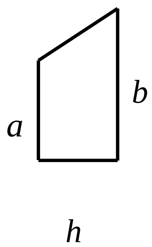

If we rotate a second trapezium through 180°, we get a parallelogram of length (a+b).

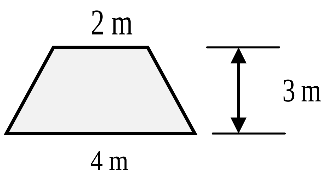
- The area of the trapezium is half the area of the parallelogram.




Alternatively, think of $\frac{a+b}{2}$ as the “average length”.

A trapezium may have two 90° angles.  The formula still works.



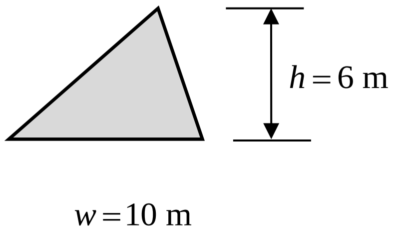

$$\text{Area } = \left(\frac{4+2}{2}\right)\times 3 = \frac{6}{2}\times 3 = 3\times 3 = 9\text{ m}^2$$

## Triangles

Word formula: “Area = base width × vertical height **÷ 2”**

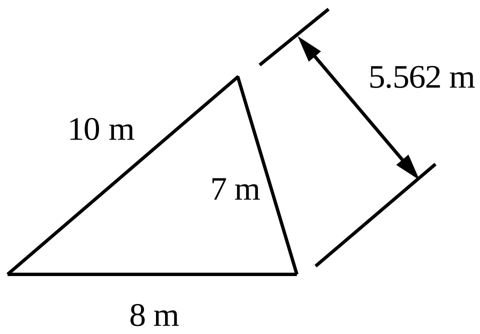
(because it is half of a parallelogram)

$A = \frac{wh}{2}$




$$A = \frac{wh}{2} = \frac{10 \times 6}{2} = \frac{60}{2} = 30 \text{ m}^2$$

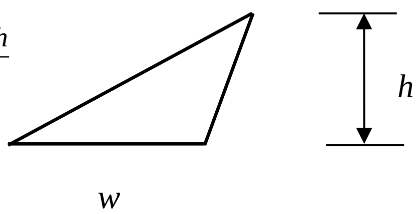
It is really important that the “vertical height” is perpendicular to the width




$$\text{Area} = \frac{10 \times 5.562}{2} = 27.81 \text{ m}^2$$

The formula **still works** if the angle is obtuse:

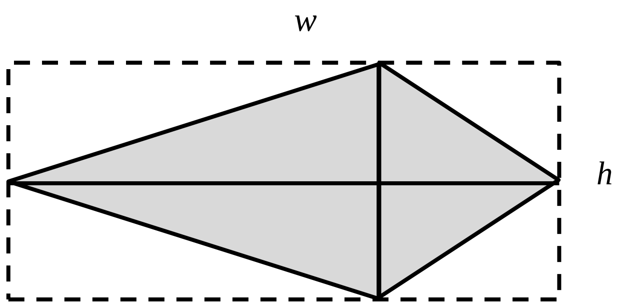
$A = \frac{wh}{2}$



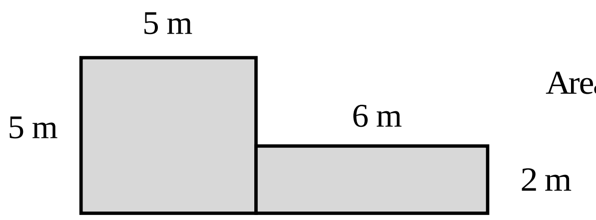

**Kites**

Word formula: “half the area of the surrounding rectangle”

$A = \frac{wh}{2}$

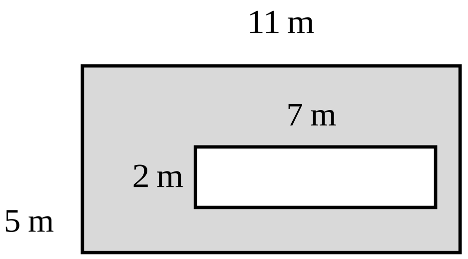


## Collections of shapes: “add the areas”:

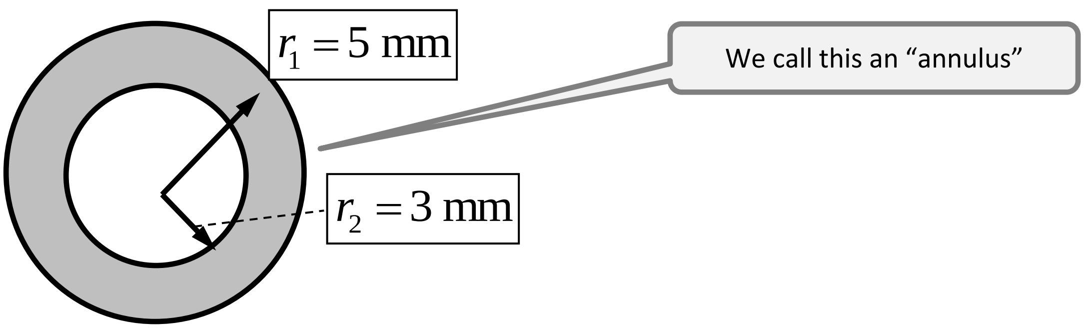



Area $= 5\times 5 + 6\times 2 = 25 + 12 = 37\ \text{m}^2$

***Shapes with holes: “subtract the missing area”:***




Shaded area $= 11\times 5 - 7\times 2 = 55 - 14 = 41\ \text{m}^2$

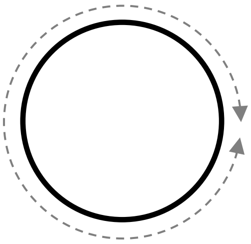



Shaded area = $\pi r_1^2 - \pi r_2^2 = \pi\left(5^2 - 3^2\right) = 50.3\ \text{mm}^2$

# Circles and parts of circles

(half a circle, a quarter of a circle etc)

***Circumference*** is the distance around the outside of the circle (for any other shape we would call this the perimeter).



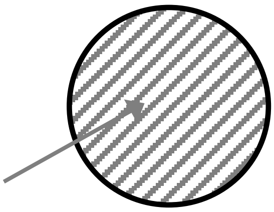
$\text{Circumference} = \pi D$

*or*

$\text{Circumference} = 2\pi r$

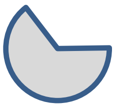
(same thing!)

$\text{Area} = \pi r^2$



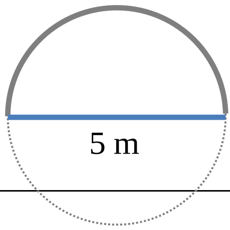
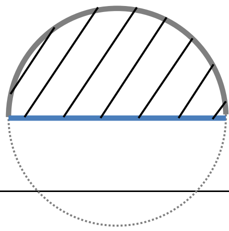
- An **arc** is a curved line, part of the circumference of a circle.

- A **sector** is a wedge of a circle, as in a pie chart



Sector questions will tell us the <u>radius</u> and the <u>angle at the centre of the circle</u>.

Just as in a pie chart, we calculate <u>what fraction of a whole circle</u> the angle represents.

| | |
|---|---|
| <u>For perimeter:</u><br>• Find the diameter D<br>• Write a formula for a fraction of the circumference $\pi D$ + 1D for the straight edges.<br>• Put numbers in the formula | <u>For area:</u><br>• Find the radius<br>• Write a formula for a fraction of the circle area $\pi r^2$<br>• Put numbers in the formula. |
| Half a circle:  |  |
| D = 5 m<br>Perimeter = $\frac{\pi D}{2} + D = \frac{\pi \times 5}{2} + 5 = 12.85 \text{ m}$ | r = 2.5 m<br>Area of <u>half</u> a circle = $\frac{\pi r^2}{2} = \frac{\pi \times 2.5^2}{2} = 19.6 \text{ m}^2$ |

Other angles?

- Just the same, work out whether it is $\frac{1}{4}$ of a circle, $\frac{1}{6}$ of a circle or whatever.

Just like a pie chart, your calculation will always be $(\text{fraction of the circle}) \times$ something

- $\text{arc length} = \left(\frac{\text{angle}}{360°}\right) \times (\pi D)$
- $\text{sector area} = \left(\frac{\text{angle}}{360°}\right) \times (\pi r^2)$
- $\text{angle} = \left(\frac{\text{arc length}}{\pi D}\right) \times 360°$
- $\text{angle} = \left(\frac{\text{sector area}}{\pi r^2}\right) \times 360°$

**e.g.** The base of a building looks like this:

The radius is 8 m. What is the area and perimeter?

Clearly this is $\frac{3}{4}$ of a circle (angle 270°, $\frac{270°}{360°} = \frac{3}{4}$)

Area = $\frac{3}{4} \times \pi r^2 = \frac{3}{4} \times \pi \times 8^2 = 48\pi = 150.8 \text{ m}^2$

Do not give answers to a ridiculous number of decimal places. 4 significant figures is plenty. Make sure you round to get the correct final digit.

Perimeter = arc length + two radiuses = $\left(\frac{3}{4} \times 2\pi r\right) + 2r = \left(\frac{3}{4} \times 16\pi\right) + 16 = 12\pi + 16 = 53.7 \text{ m}$

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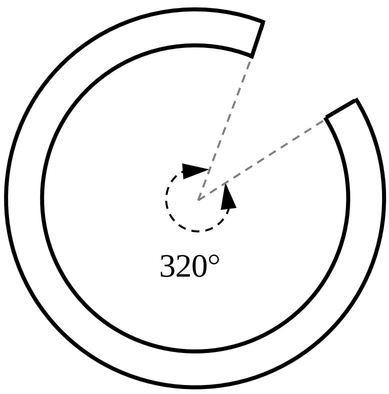
**e.g.** A metal ring has a gap cut in it. The inner radius is 10 cm, the outer radius is 13 cm. and the included angle is 320°. What is the area remaining?




Area = (fraction of 360°) × (area of big circle – area of small circle)

$$= \left(\frac{320°}{360°}\right)(\pi \times 13^2 - \pi \times 10^2) = \frac{8}{9}\pi(169 - 100) = 192.7 \text{ cm}^2$$

## Problems where a length must be found

- Decide which of the circumference, area of volume formulae you need
  - *If unsure, I suggest you sketch the shape as you read the question, marking the dimensions on your sketch*
- Write an equation in the form "formula = value"
- Solve the equation

Examples:

(a) An explorer walks around the rim of the Pingualit crater

He counts his paces and estimates the circumference is 10.8 km. What is the diameter?

$$C = \pi D = 10.8$$

$$D = \frac{10.8}{\pi} = 3.44 \text{ km}$$

(b) An engine designer wants to make a piston (a cylindrical plug that slides in the cylinder of an engine) with a cross-sectional area of 0.3 $m^2$. What diameter should it be?

Area of a circle is $A = \pi r^2$ **(formula)**

**= 0.3 $m^2$ (value)**

Combine the two, "formula = value":

$\pi r^2 = 0.3$ **(equation)**

$r^2 = \frac{0.3}{\pi}$ on calculator, then

$r = \sqrt{ANS} = 0.309 \text{ m}$

$$D = 2 \times ANS = 0.618 \text{ m}$$

(c) A cylinder has radius 5 m and volume 200 $m^3$. Find the length.

$\pi r^2 L = 200$ **(equation)**

$$L = \frac{200}{\pi \times 5^2} = 2.546 \text{ m}$$

(d) A cylinder has volume 20 $m^3$ and length 3 m. Find the radius.

$\pi r^2 L = 20$ **(equation)**

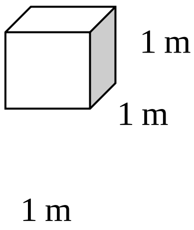
$r^2 = \frac{20}{\pi L} = \frac{20}{3\pi}$ on calculator, then $r = \sqrt{ANS} = 1.457 \text{ m}$

## Surface area

**3D shapes.**

**Surface area**: add up the area of each surface.

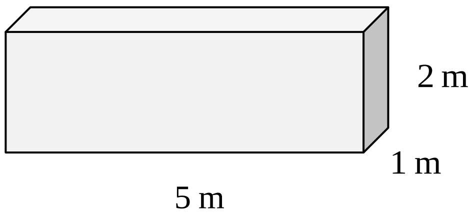
Cube with 1 m sides:




Each face has area 1m × 1m = $1m^2$.

The cube has 6 faces (like a dice) so the total surface area = $6m^2$.

Cuboid:




Add up all the areas:

Front area = 5×2 = 10 $m^2$, front + back = 10+10 = 20 $m^2$.

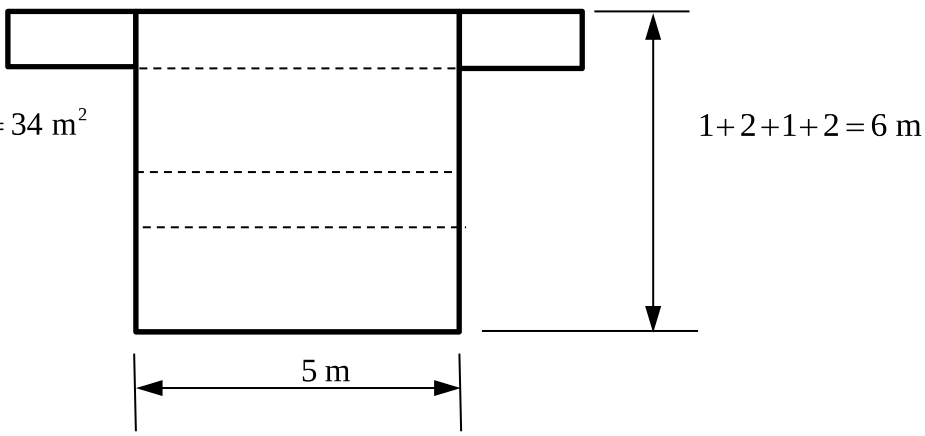
Top area = 5×1 = 5 $m^2$, top + bottom = 5+5 = 10 $m^2$

End area = 1×2 = 2 $m^2$, left end + right end = 2+2 = 4 $m^2$.

Total 20 + 10 + 4 = 34 $m^2$

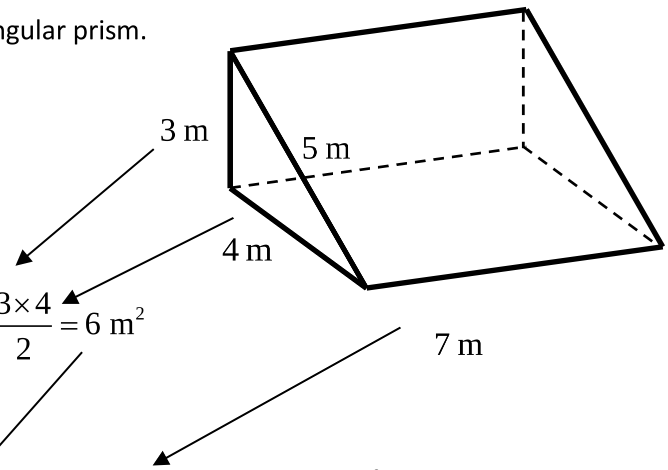
or, unwrap it to make a net:

$$\text{Area} = 5\times 6+2(1\times 2)=34 \text{ m}^2$$



Examples

(1) Find the surface area of this triangular prism.




Cross-section area = $\frac{wh}{2} = \frac{3\times 4}{2} = 6 \text{ m}^2$

Surface area = 2×6 + (3+4+5) ×7 = 12 + 12×7 = 96 $m^2$.

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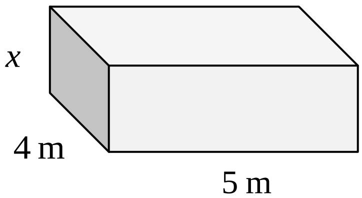
(2) The surface area of this cuboid = 112 $cm^2$. Find the volume.

First we must find the height x.




Surface area = $2(4\times 5)+(4+5+4+5)x = 40+18x$

Hence we need $40+18x=112$ **(equation)**

$18x = 112-40 = 72$

$x = \frac{72}{18} = 4 \text{ m}$

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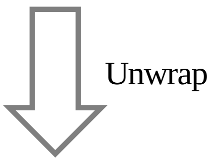
(3) Surface area of a cylinder.

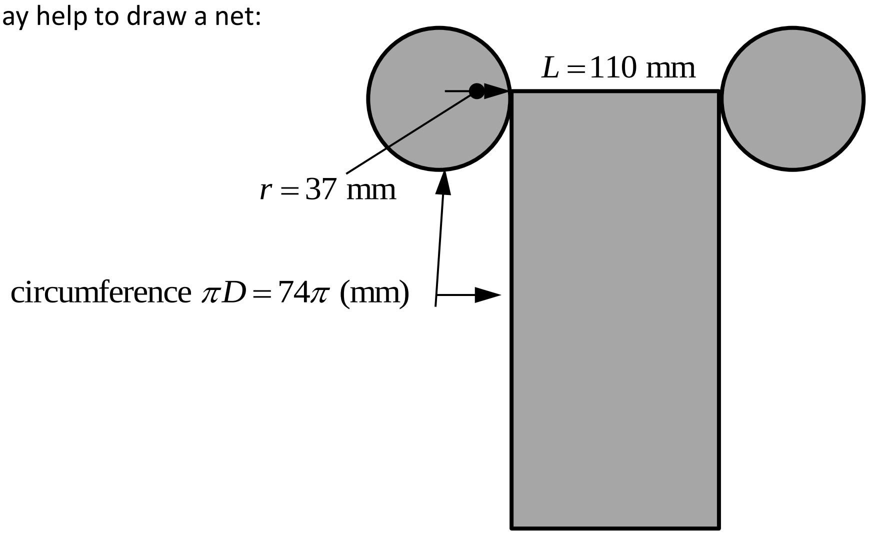
- Always ask yourself: "Is it a closed container (two circular ends) or an open one (one end)?"

A baked bean tin is 74mm diameter and 110 mm long.

Find its surface area.

It may help to draw a net:




Total area = $2\left(\pi r^2\right) + \pi DL = 2\pi \times 37^2 + \pi \times 74 \times 110$

= 34174 mm$^2$ (=342 cm$^2$ to 3 significant figures)

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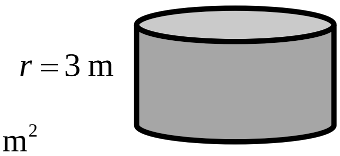
**(4) <u>Surface area of *half* a cylinder</u>**

**(i) Diameter D=6m, height h=5 m**




Whole cylinder surface area $2\left(\pi r^2\right) + 2\pi rh = 2\pi\left(r^2 + rh\right) = 150.8\ \text{m}^2$

(ii) <u>Half of this cylinder, sliced vertically</u>

Surface area $2\left(\frac{1}{2}\pi r^2\right) + \pi rh + 2rh = \pi\left(r^2 + rh\right) + 2rh = 105.4\ \text{m}^2$

(half the whole cylinder <u>plus the cut surface</u> (rectangle 2r×h)

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# Spheres

Surface area = $4\pi r^2$

**e.g**. A spherical balloon has a radius of 10 m.

What is its surface area?

$$A = 4\pi r^2 = 4\pi \times 10^2 = 400\pi = 1260 \text{ m}^2$$

(to 3 sig. figures)

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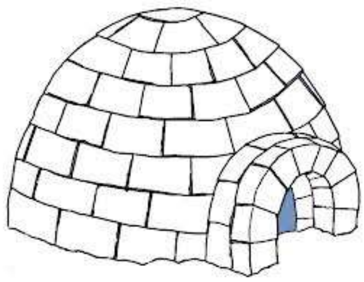
**e.g**. An igloo is a hemisphere of diameter 3 m. What is its surface area?



$$r = \frac{D}{2} = 1.5 \text{ m}$$

Area = $\frac{1}{2}\left(4\pi r^2\right) = 2\pi \times 1.5^2 = 14.1 \text{ m}^2$

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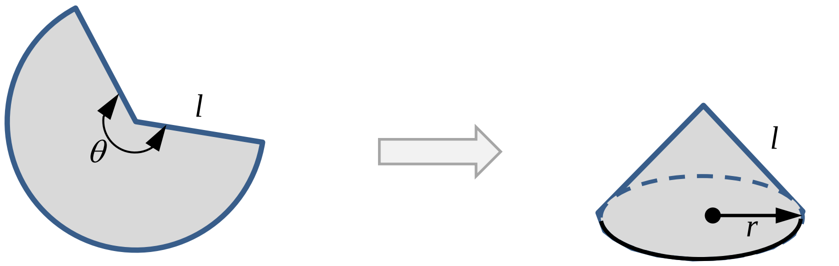
## Cones

If a sector of a circle is rolled up, it makes the curved surface of a cone:




sector area $\left(\frac{\cancel{2}\pi r}{\cancel{2}\pi l}\right)\pi l^2 = \pi r l$

base circumference $2\pi r$

(fraction of circle) × (whole circle area)

Curved surface area = $\pi r l$

We can also calculate the “included angle” $\theta$ (“theta”):

We can see that sector arc length $\left(\frac{\theta}{360}\right)2\pi l =$ cone base circumference, $2\pi r$

Therefore $\frac{\theta}{360} = \frac{r}{l}$

**Example**

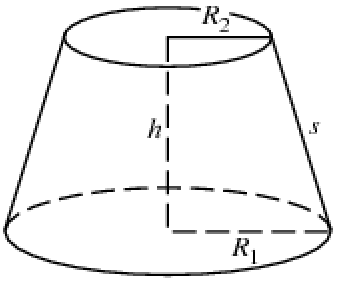
I want to make a witch’s hat (image from wpclipart.com):

The conical part is to be 40 cm tall and 22 cm diameter at the base. What is its surface area?

$\text{area} = \pi r l = \pi \times 11 \times 40 = 1380 \text{ cm}^2$

## The frustrum of a cone (extension, A* only)

A frustrum is a cone with its tip cut off.

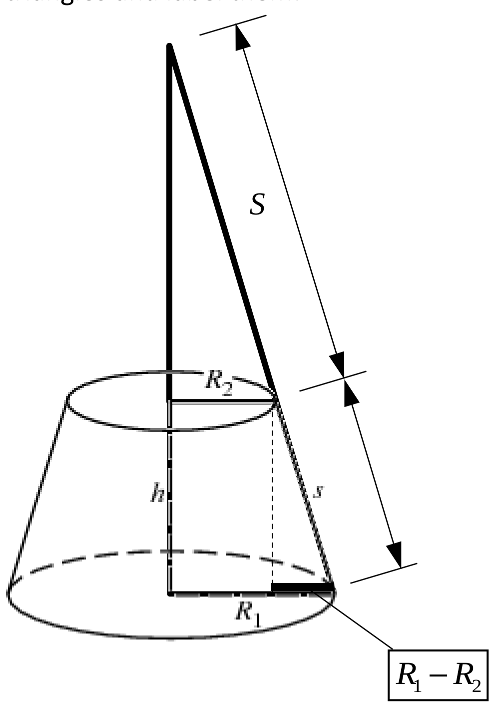



(image from Wolfram MathWorld – have a look!). This would be a difficult A* question!

Just as we can subtract a small circle from a larger one to find the area of an annulus, we can subtract a small cone from a larger one to find the area of the "bucket shape" that's left over.

You have the base radius for the two cones - the difficulty is that you need to find the slant height for each.

You need to draw a pair of similar triangles and label them:




Since the triangles are similar, the hypotenuses are parallel (they have the same gradient)

$$\frac{S}{r_2} = \frac{s}{r_1 - r_2}$$

Hence $S = \frac{r_2}{r_1 - r_2} s$, we calculate a value for *S*.

Knowing that the surface area of a cone is $\pi r l$, we say:

“frustrum surface area = area of big cone – area of small cone “

$$= \pi R_1 (S + s) - \pi R_2 S$$

*<u>If you got a question like this, you would probably find that the numbers were chosen so you could just spot the size of the imaginary “upper cone”.</u>*

<u>Example</u>

A buoy is 1.6 m tall (slope height). It is 40 cm diameter at the base and 20 cm diameter at the top.

Find the area of the curved surface.

Since the bottom is double the diameter of the top, what we see is the <u>bottom half</u> of a cone 2×1.6 = 3.2 m high

Curved surface area = (area of whole cone) – (area of top section)

$$= \pi \times 0.2 \times 3.2 - \pi \times 0.1 \times 1.6$$

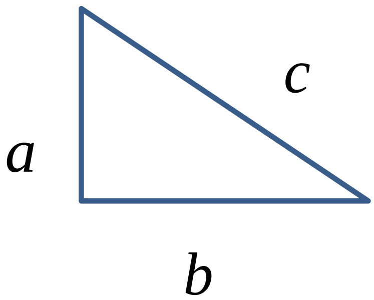
remember to use radius not diameter

= 1.51 $m^2$.

## Problems requiring Pythagoras (A* topic)

Some questions with pyramids or cones require us to use Pythagoras to find the slant height before we go on to find the surface area.

- Draw yourself a right angled triangle and mark its sides a, b and c




- To find the hypotenuse, use $c^2 = a^2 + b^2$
- To find one of the shorter sides, use $a^2 = c^2 - b^2$ (or $b^2 = c^2 - a^2$)