Locus that is a constant distance from a line

If the line is horizontal or vertical, this is very easy.

e.g. Define the locus of all points a distance of 2.5 units from (a) y = 1, (b) x = 2

(a). We need two horizontal lines y = 1+2.5 = 3.5 and y = 1-2.5 = -1.5

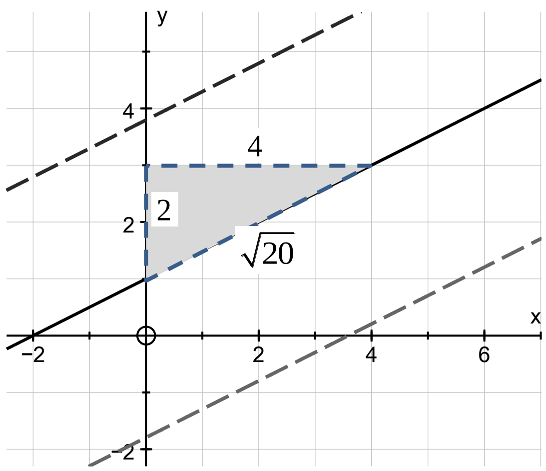
(b) We need two vertical lines x = 2+2.5 = 4.5 and x = 2-2.5 = -0.5

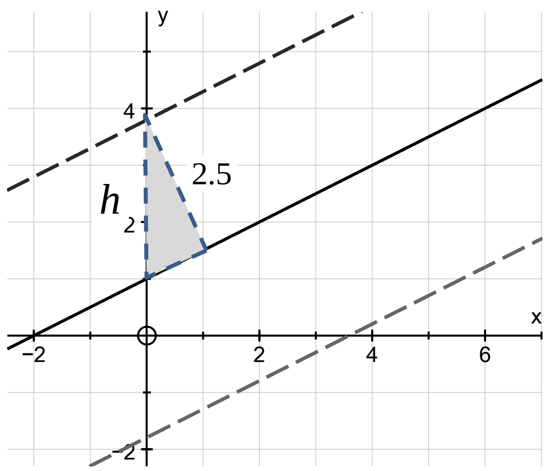
For a sloping line y = mx+c, the amount we add or subtract from "c" is more than the line spacing. There are 2 ways of thinking about this:

- similar triangles (proportion)
- trigonometry

Example: Define the locus of all points a distance of 2.5 units from line $y = \frac{1}{2}x + 1$

(a) similar triangles "positive gradient, triangle above line, hold the bottom corner and flip"



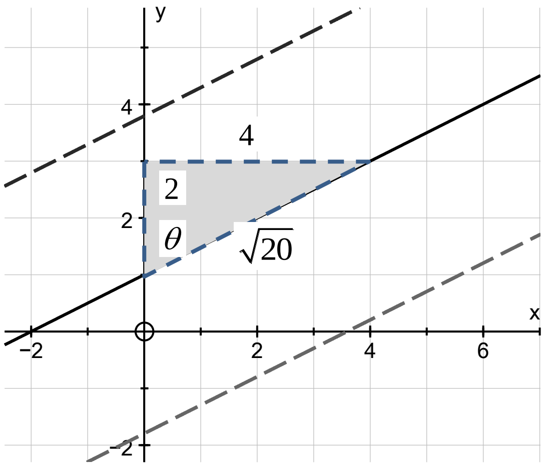


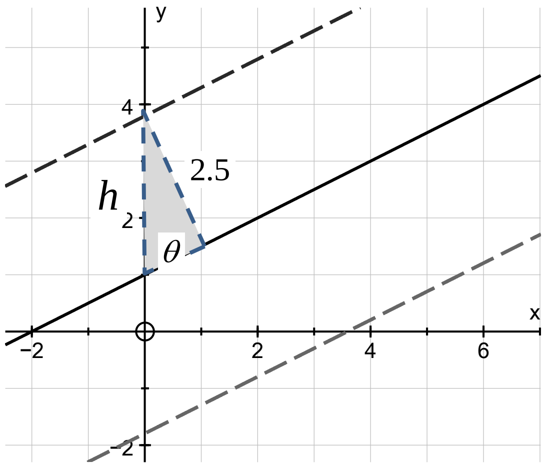
The triangles are similar hence $\frac{h}{2.5} = \frac{\sqrt{20}}{4}$

We need $h = \frac{2.5 \times \sqrt{20}}{4} = 2.795$

My new lines are $y = \frac{1}{2}x + 1 + 2.795$ and $y = \frac{1}{2}x + 1 - 2.795$

Alternatively, thinking in terms of trigonometry:



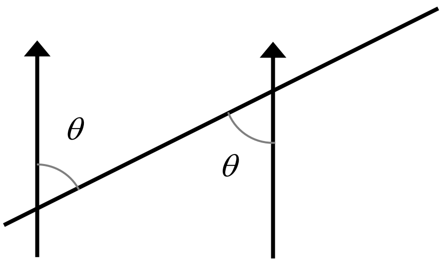


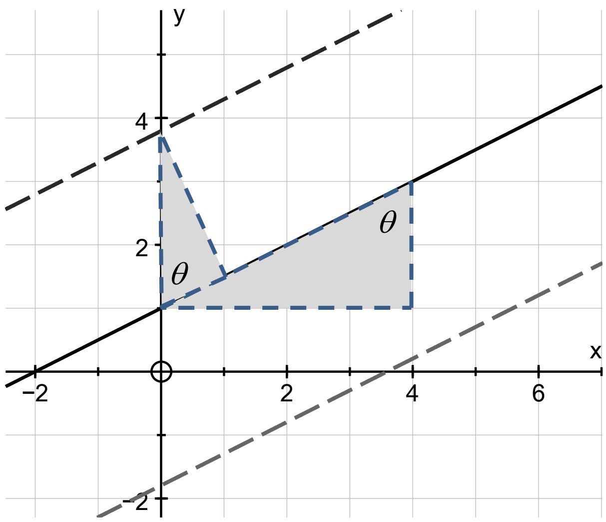
The line spacing is $h \times \sin\theta = 2.5$, hence we need $h = \dfrac{2.5}{\sin\theta}$

We know from Pythagoras that the hypotenuse in the first graph is $\sqrt{20}$ hence $\sin\theta = \dfrac{4}{\sqrt{20}}$. Get this as ANS, then $h = \dfrac{2.5}{ANS} = 2.795$

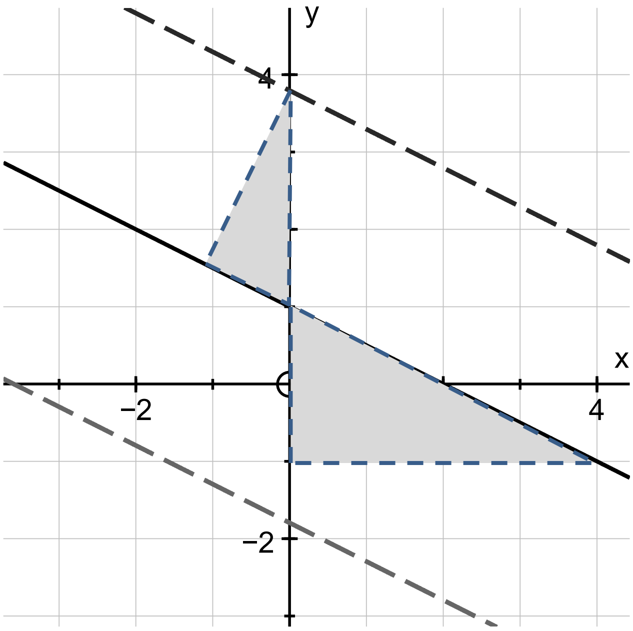
If you cannot remember which way to draw the two triangles, don't panic!

- You will find that spotting *parallel vertical lines* helps you identify the equal angles θ (and then *which two* sides to use in the ratio):





"Negative gradient, vertically opposite angles from top corner are equal":



- same process!