Surds

Any root (square root, cube root etc) that is irrational is a *surd*.

[Meaning that some roots e.g. $\sqrt{9} = 3$, $\sqrt{2\tfrac{1}{4}} = 1\tfrac{1}{2}$ are *integers* or *rational numbers* but most roots cannot be accurately written down as decimals or any other kind of fraction – we call these *surds*].

$\sqrt{2} \approx$
1.4142135623730950488016887242096980785696718753769480731766797379907324784621

- It would get tiresome writing down all these figures
- Even then, if we square it we do not quite get 2 back again (because it is not exact)
- It is not obvious that it means $\sqrt{2}$

Instead we just write $\sqrt{2}$ and then we all know what we mean. Surds often occur in the solution of equations.

Rules.

1. $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{\left(a^n\right)} = a$ "by definition!"

   So $\left(\sqrt{5}\right)^2 = 5$, $\left(\sqrt[3]{11}\right)^3 = 11$, $\left(\sqrt[5]{17}\right)^5 = 17$ etc.

   Similarly $\sqrt{25} = \sqrt{5^2} = 5$, $\sqrt[3]{1331} = \sqrt[3]{11^3} = 11$ etc.

2. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

   Hence $\sqrt{10} = \sqrt{2 \times 5} = \sqrt{2}\sqrt{5}$

   Note: $\sqrt{2}\sqrt{5}$ means $\sqrt{2} \times \sqrt{5}$ just like 2π means $2 \times \pi$
   [so unlike $1\tfrac{1}{2} = 1 + \tfrac{1}{2}$, it does **not** mean $\sqrt{2} + \sqrt{5}$ ***!!!!!***]

3. $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

   Hence $\sqrt{\dfrac{9}{4}} = \dfrac{\sqrt{9}}{\sqrt{4}} = \dfrac{3}{2}$

4. $a\sqrt{b} + c\sqrt{b} = (a+c)\sqrt{b}$

   Hence $2\sqrt{7} + 4\sqrt{7} = 6\sqrt{7}$

Simplifying surds.

Example. Evaluate: (a) $\sqrt[3]{\dfrac{8}{125}}$ (b) $\sqrt{0.0016}$

$$\therefore \sqrt[3]{\frac{8}{125}} = \frac{\sqrt[3]{8}}{\sqrt[3]{125}}$$

$$= \frac{2}{5}$$

$$\therefore \sqrt{0.0016} = \sqrt{\frac{16}{10000}}$$

$$= \frac{4}{100} \text{ or } \frac{1}{25} \text{ or } 0.04$$

Example. Simplify $\sqrt{18}$

$$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$$

Example. Simplify $\frac{\sqrt{45}}{3}$

$$\frac{\sqrt{45}}{3} = \frac{\sqrt{9 \times 5}}{3} = \frac{\sqrt{9}\sqrt{5}}{3} = \frac{3\sqrt{5}}{3} = \sqrt{5}$$

Nb. Note the difference between $3\sqrt{5}$ and $\sqrt[3]{5}$ (a cube root).

2-stage problems.

We might need to simplify a surd expression and add it to another.

- We can only simplify the sum of two surds if they can be simplified to the same surd - we can simplify $\sqrt{3} + \sqrt{3}$ but we cannot simplify $\sqrt{2} + \sqrt{3}$.

Example.

Simplify: $\sqrt{75} + \sqrt{12}$ expressing your answer in the form $k\sqrt{3}$

First we simplify each surd to see if they are multiples of a simpler one:

$$\sqrt{75} + \sqrt{12} = \sqrt{25 \times 3} + \sqrt{4 \times 3}$$

$$= 5\sqrt{3} + 2\sqrt{3}$$

Now we can add them: $= 7\sqrt{3}$

Multiplying brackets containing surds.

Remember that to multiply out brackets that add terms, we need to add all the cross-products.

Eg. $(10+2)(4+1) = 10\times4 + 2\times4 + 10\times1 + 2\times1$

$$= 40 + 8 + 10 + 2 = 60$$

(Remember *FOIL: First, Inner, Outer, Last*)

The process is the same even if some of the numbers are surds.

Example. Simplify $\left(3+2\sqrt{5}\right)\left(1-\sqrt{5}\right)$

$$\left(3+2\sqrt{5}\right)\left(1-\sqrt{5}\right) = 3 - 3\sqrt{5} + 2\sqrt{5} - 2\sqrt{5}\sqrt{5}$$

This process of adding the rational number parts and then adding up the similar surd parts is known as "***collecting terms***"

$$= 3 - \sqrt{5} - 10 = -7 - \sqrt{5}$$

Example. Multiply out and simplify the following: $\left(4-\sqrt{5}\right)\left(3+\sqrt{2}\right)$.

$$\therefore \left(4-\sqrt{5}\right)\left(3+\sqrt{2}\right) = 12 + 4\sqrt{2} - 3\sqrt{5} - \sqrt{5}.\sqrt{2}$$

$$= 12 + 4\sqrt{2} - 3\sqrt{5} - \sqrt{10}$$

Rationalising a denominator.

Rules:

- To simplify a fraction like $\frac{b}{\sqrt{a}}$, multiply by $\frac{\sqrt{a}}{\sqrt{a}}$
- To simplify a fraction like $\frac{c}{a\pm\sqrt{b}}$, multiply by $\frac{a\mp\sqrt{b}}{a\mp\sqrt{b}}$ (always need one + and one -).

Nb. You are multiplying by 1 so the value does not change.

Example.

Rationalise the denominator of $\frac{2}{\sqrt{3}}$.

$$\therefore \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\therefore \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

<u>Example</u>.

Rationalise the denominator: $\frac{3\sqrt{3}}{5-\sqrt{2}}$

$$\frac{3\sqrt{3}}{5-\sqrt{2}} = \frac{3\sqrt{3}}{5-\sqrt{2}} \times \frac{(5+\sqrt{2})}{(5+\sqrt{2})}$$

(now the denominator is a "difference of two squares")

$$= \frac{15\sqrt{3}+3\sqrt{3}.\sqrt{2}}{25-5\sqrt{2}+5\sqrt{2}-\sqrt{2}.\sqrt{2}}$$

$$= \frac{15\sqrt{3}+3\sqrt{6}}{25-2}$$

$$= \frac{15\sqrt{3}+3\sqrt{6}}{23}$$