

Ways of finding a probability

1. If you have a number of equally likely outcomes, you can find the probability

exactly.
$$\text{probability} = \frac{\text{number of "successful" outcomes}}{\text{number of possible outcomes}}$$

e.g. the probability of throwing a 1 or a 2 on a dice, you have 2 ways of being successful out of 6 possibilities, each face has a probability of $\frac{1}{6}$ so $P(1 \text{ or } 2) = \frac{2}{6} = \frac{1}{3}$

2. If you cannot use "equally likely outcomes" you need to do a series of experiments ("trials"). These give you a *relative frequency* that is an *estimate of the probability*.

The more trials you do, the better the estimate.

$$\text{relative frequency} = \text{estimated probability} = \frac{\text{number of "successful" outcomes}}{\text{number of trials}}$$

3. If you cannot do an experiment, you need to do the same calculation but using data that already exists, e.g. aircraft accident data for the last 10 years.

4. Often, politicians will just guess a probability. This can be wildly inaccurate (e.g. the space shuttle was thought to have 1 in 10,000 risk of disaster when it was probably about 1 in 100).

Relative frequency examples.

Throwing dice, using Excel as a random number generator (=ROUND(RAND()*6+0.5, 0) will generate an integer in the range 1 to 6).

Number of throws	6	24	120	6000
Number of ones	0	3	17	972
Number of twos	1	3	27	1054
Number of threes	2	3	19	980
Number of fours	1	7	25	1041
Number of fives	1	5	12	975
Number of sixes	1	3	20	978
Expected frequency	1	4	20	1000

We calculate:

$\text{expected frequency} = \text{probability} \times (\text{number of trials})$

You can see that the random variations get smoothed out as we do more and more trials, giving a better estimate of the probability.

Simulation.

Imagine running a restaurant. 30 people come to order breakfast. Usually, 1 in 10 people wants porridge. We can calculate an expected frequency $\frac{1}{10} \times 30 = 3$ people who should want it.

Of course, the number is random. Sometimes it will be more than 3, sometimes less. (3 is just the average over many days). Will the chef sometimes need to make 6 lots of porridge? or 10? or 29?

- If you were doing A-level Maths you would have a big formula for calculating this. At gcse, we do a simulation to get some feel for how likely it is. (And in real life, one gets situations that need simulation because they are too complicated for a formula).

To do a simulation you need a random number generator that will simulate the probability you want, e.g. a coin, a die, the RAN# button in your calculator or the =RAND() function in Excel.

RAN# generates numbers 0, 0.001, 0.002...up to 0.999. You can use groups of these to get probabilities that are multiples of $\frac{1}{1000}$ e.g. to get $\frac{250}{1000} = \frac{1}{4}$ you would use the first 250 numbers, 0.000 up to 0.249.

Questions often use integers from 0 to 99. You can get these just by reading the first 2 digits, e.g. look at 0.247 and call it 24.

- So with the porridge, to get $\frac{1}{10}$ we would look for numbers 0.000 to 0.099. We could push RAN# thirty times and see if we got 6 like this, or more than 10, or more than 29. (Alternatively, look for any number starting with 1 e.g. 0.129)
- We would need to repeat the process lots of times to get a good impression of what might happen.
- It is a lot easier doing this in Excel!

Looking at a lottery, what are the chances of balls 1, 2, 3, 4, 5, 6 coming up? Is it worth playing? Does the number of balls matter?

Doing a simulation with different numbers of balls, I play 100,000 games. The number of times I win is:

Number of balls	7	10	15	20	49
Number of wins	14286	485	27	1	0