Simplifying algebraic fractions

Number examples

We can convert a fraction to its equivalent by multiplying top and bottom by the same thing

$$\frac{1}{2}=\frac{2}{4}=\frac{20}{40}$$

For a simpler equivalent fraction, we divide top and bottom by a common factor

$$\frac{30}{50}=\frac{3}{5}$$

$$\frac{48}{8}=\frac{6}{1}=6$$ (same as 48÷8=6)

We can divide by letters instead of numbers:

$$\frac{3x}{5x}=\frac{3}{5}$$

Remember rules of indices: $\frac{x^2}{x}=x$, $\frac{x^3}{x}=x^2$, in general $\frac{x^m}{x^n}=x^{m-n}$

$$\frac{24x^5}{6x^2}=\frac{4x^5}{x^2}=\frac{4x^3}{1}=4x^3$$

$$\frac{(x-2)^2}{(x-2)}=(x-2)$$

$$\frac{12x}{4x^2}=\frac{3}{x}$$

$$\frac{35x^4y}{7x^2y^2}=\frac{5x^2}{y}$$

$$\frac{24x^3-16x^2}{8x}=\frac{3x^3-2x^2}{x}=3x^2-2x$$

Harder questions require factorising before we can simplify

Factorise to find factors that might cancel:

$$\frac{3x+6}{x+2}=\frac{3(x+2)}{(x+2)}=3$$

$$\frac{x^2+2x}{x+2}=\frac{x(x+2)}{(x+2)}=x$$

$$\frac{15-3x}{x-5}=\frac{3(5-x)}{x-5}=\frac{-3(x-5)}{(x-5)}=-3$$

If you have a quadratic it is probably easier to factorise in a separate "thinking box":

"Simplify $\dfrac{x^2+2x-3}{x-1}$"

Factorise x^2+2x-3

Factors of -3 are 1×-3 (adds to -2, no good)

-1×3 (adds to +2, YES!!!!)

$\therefore \quad x^2+2x-3=(x-1)(x+3)$

Hence $\dfrac{x^2+2x-3}{x-1}=\dfrac{(x-1)(x+3)}{x-1}=x+3$

Simplify $\dfrac{x^2+13x+36}{x+9}$

Factorise $x^2+13x+36$

Factors of 36 are 1×36 (adds to 37, no good)

2×18 (adds to 20, no good)

3×12 (adds to 15, no good)

4×9 (adds to 13, YES!!!)

6×6 (adds to 12, no good)

and the negative equivalents.

$\therefore \quad x^2+13x+36=(x+4)(x+9)$

$\dfrac{x^2+13x+36}{x+9}=\dfrac{(x+4)(x+9)}{x+9}=x+4$

Simplify $\dfrac{7x^2-9x-10}{x-2}$

Factorise $7x^2-9x-10$

ac = 7×(-10) = -70

Factors of -70 are 1×-70 (adds to -69, no good)

2×-35 (adds to -33, no good)

5×-14 (adds to -9, YES!!!)

7×-10 (adds to -3, no good)

10×-7 (adds to 3, no good)

etc

$\therefore \quad 7x^2-9x-10=\left(7x+\dfrac{5}{1}\right)\left(1x-\dfrac{14}{7}\right)=(7x+5)(x-2)$

Check:

$(7x+5)(x-2)=7x^2-14x+5x-10=7x^2-9x-10$

$\dfrac{7x^2-9x-10}{x-2}=\dfrac{(7x+5)(x-2)}{x-2}=7x-5$