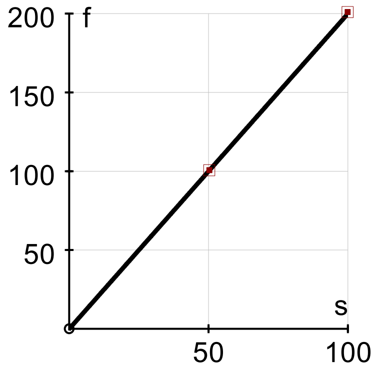
Proportion

Two quantities are said to be in *direct proportion* when the ratio of one to the other is constant. We can also say that one is proportional to the other.

For instance, a recipe for syrup pudding includes 100g flour and 50g sugar. Regardless of how many puddings we make (multiples of the recipe), we know the amount of flour ***f*** is proportional to the amount of sugar ***s***. If one doubles, so must the other.

Instead of an = sign, we have a special sign to indicate "is proportional to": $f \propto s$

This implies that the graph of ***f*** against ***s*** would be a straight-line through the origin with some gradient ***k*** (equation $f = ks$):



Typically a question will give you a pair of coordinates on the line. This tells us the gradient of the line.

e.g. A recipe includes 100g flour and 50g sugar.
How much flour will I need if I am going to use 100g sugar?

There are 3 ways of thinking about this – they are all good, but one approach is particularly good as it lets us do more complicated problems later.
(1) Getting the formula by finding the gradient.

$\text{gradient } k = \frac{\text{up}}{\text{across}} = \frac{100}{50} = 2$, hence the formula is $f = 2s$.
Then put $s = 100$ into the formula to get $f = 2 \times 100\text{g} = 200\text{g}$
nb (*nota bene*): be careful with the units of your answer!

(2) the "unitary" method: how much flour do we need for 1 gram of sugar?
For 1 g sugar I need $\frac{100\text{g}}{50} = 2\text{g}$ flour, so then for 100 g sugar I need $2\text{g} \times 100 = 200\text{g}$ flour.
Hey! That's the same as the gradient method, I just didn't call it gradient!

(3) The "scaling up" or "enlargement factor" method. (**best way**!)
What multiple of the recipe am I making?
I am making $\frac{100}{50} = 2\times$ the recipe so I need $2 \times 100\text{g} = 200\text{g flour}$.

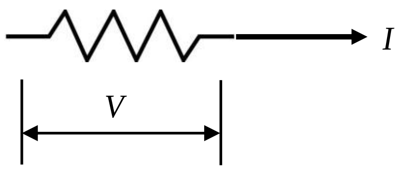
$\left(\frac{100}{50}\right)$ is the "enlargement factor".

<u>Examples</u>
(a) the power a solar cell can produce is proportional to its surface area.
A cell with 0.3 m^2 area produces 15 Watts.

- What area will I need to generate 300 Watts?
  - If I want to generate 300 Watts, I will need $\frac{300\text{ W}}{15\text{ W}}\times 0.3\text{ m}^2 = 6\text{ m}^2$
- If I have 24 m^2, what power will it generate?
  - $\frac{24\text{ m}^2}{0.3\text{ m}^2}\times 15\text{ W} = 1200\text{ W}$

Note: The really nice thing about this method is that the enlargement factors $\frac{300\text{ W}}{15\text{ W}}$ or $\frac{24\text{ m}^2}{0.3\text{ m}^2}$ are <u>dimensionless</u> (just a number, no units). The Watts or m^2 cancel out top and bottom. We then automatically keep the correct units when we scale up the 0.3 m^2 or the 15 W.

(b) The current *I* through a resistor is proportional to the voltage *V* across it.



When *V* = 20 volts, *I* = 4 amps.

Because we know about proportion, you do not need to have learnt about voltage and current – it is "just a proportion question".

What is the current *I* if *V* = 240 Volts?

$I = \frac{240\text{ volts}}{20\text{ volts}}\times 4\text{ amps} = 48\text{ amps}$ ("12× the voltage so 12× the current")

What is the voltage if the current is 1 amp?

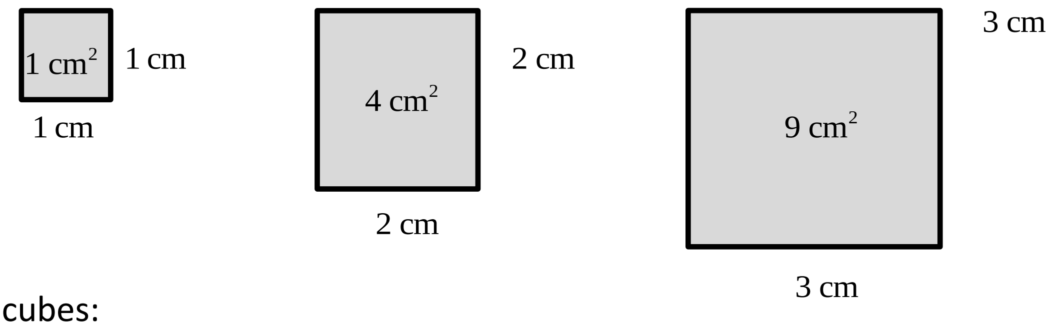
$V = \frac{1\text{ amp}}{4\text{ amps}}\times 20\text{ volts} = 5\text{ volts}$ ("¼ the current so ¼ the voltage")

Area and volume scaling

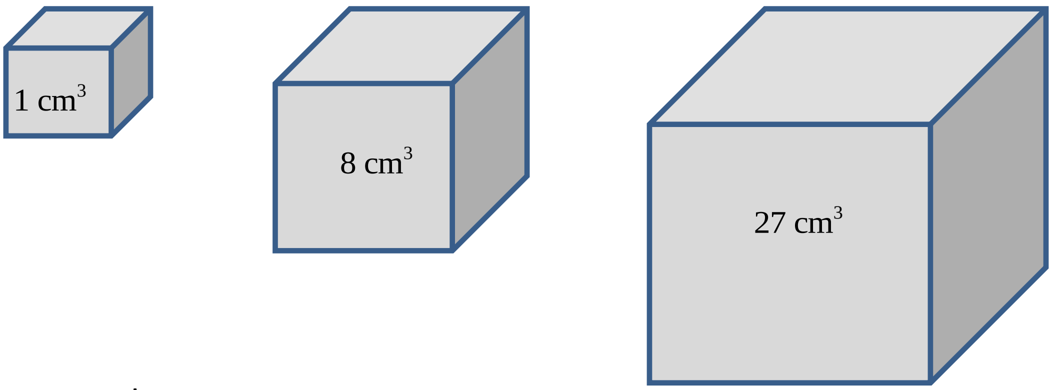
When an object is enlarged to make a new object that is "geometrically similar" (all angles same as before, all lengths in the same proportion):

- $\text{area} \propto (\text{enlargement factor})^2$
- $\text{volume} \propto (\text{enlargement factor})^3$

e.g. squares:



cubes:



Remember:

- $100 \text{ cm} \times 100 \text{ cm} = 10000 \text{ cm}^2 = 1 \text{ m}^2$
- $1000 \text{ m} \times 1000 \text{ m} = 1000000 \text{ m}^2 = 1 \text{ km}^2$
- $10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} = 1000 \text{ mm}^3 = 1 \text{ cm}^3$

Examples:

(i) A drawing of a garden shows a pond, area 12 cm^2 on the paper. The scale of the drawing is 1:50. What is the area of the pond?

Area = $50^2 \times 12 \text{ cm}^2 = 30000 \text{ cm}^2 = 3 \text{ m}^2$.

(ii) A ship 20 m long has has a volume of 150 m^3. A bigger version of the ship is 30 m long. What is its volume?

volume = $\left(\frac{30 \text{ m}}{20 \text{ m}}\right)^3 \times 150 \text{ m}^3 = 1.5^3 \times 150 \text{ m}^3 = 506.25 \text{ m}^3$