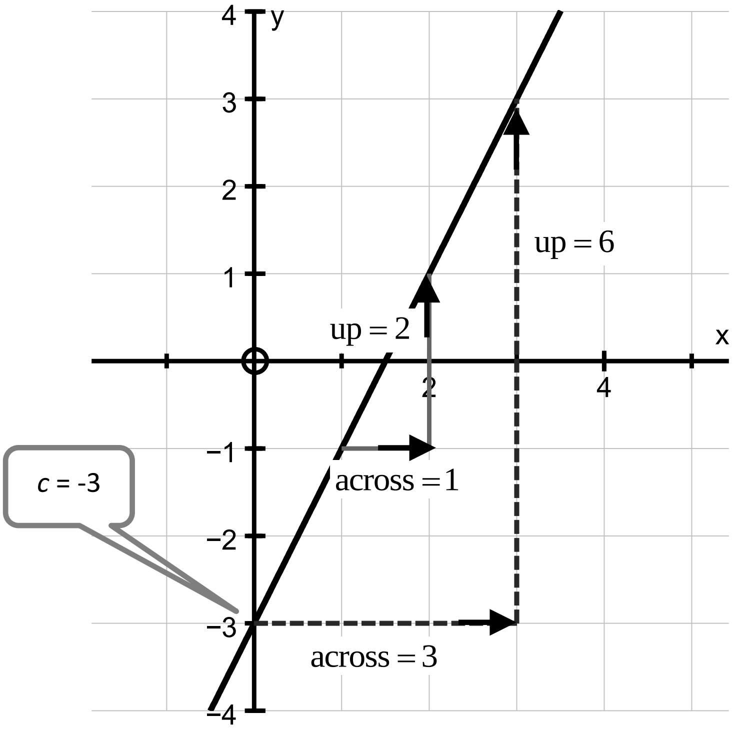
Line gradients, y=mx+c, parallel and perpendicular lines.

The gradient of a line tells us how much it goes up for every 1 unit you move across.

- Positive gradient is a slope ↗
- A horizontal line (e.g. y=2) → has zero gradient.
- Negative gradient is a slope ↘
- A vertical line (e.g. x=1) ↑ has infinite gradient.

The equation $y = mx + c$ has a number ***m*** which is the gradient and a number **c** which is the "y-intercept", the y-value corresponding to the *x*=0 point where the line cuts the *y*-axis.

The gradient is calculated as $m = \frac{\text{up}}{\text{across}}$



All "across, up" triangles are similar and give the same gradient:

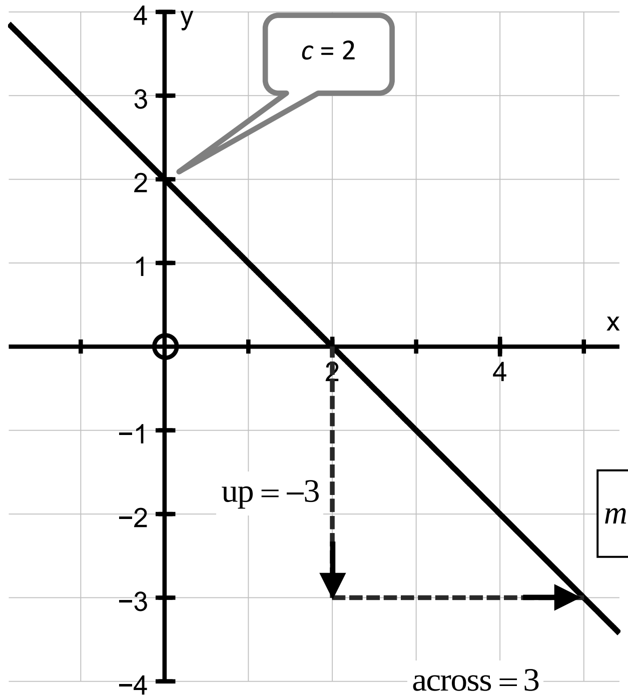
$m = \frac{2}{1} = 2$, or

$m = \frac{6}{3} = 2$

The equation of this line is:
$y = 2x - 3$

The arrows must be drawn "nose to tail".

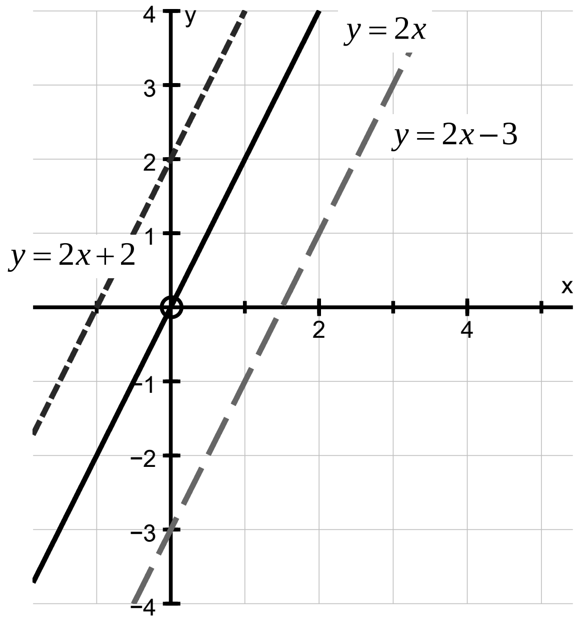
If the line slopes downwards, *either* the "across" *or* the "up" value will be negative, so the gradient is negative:



The equation of this line is:
$y = -x + 2$

$m = \frac{-3}{3} = -1$

Parallel lines all have the same gradient, for instance these lines are parallel because they all have gradient =2

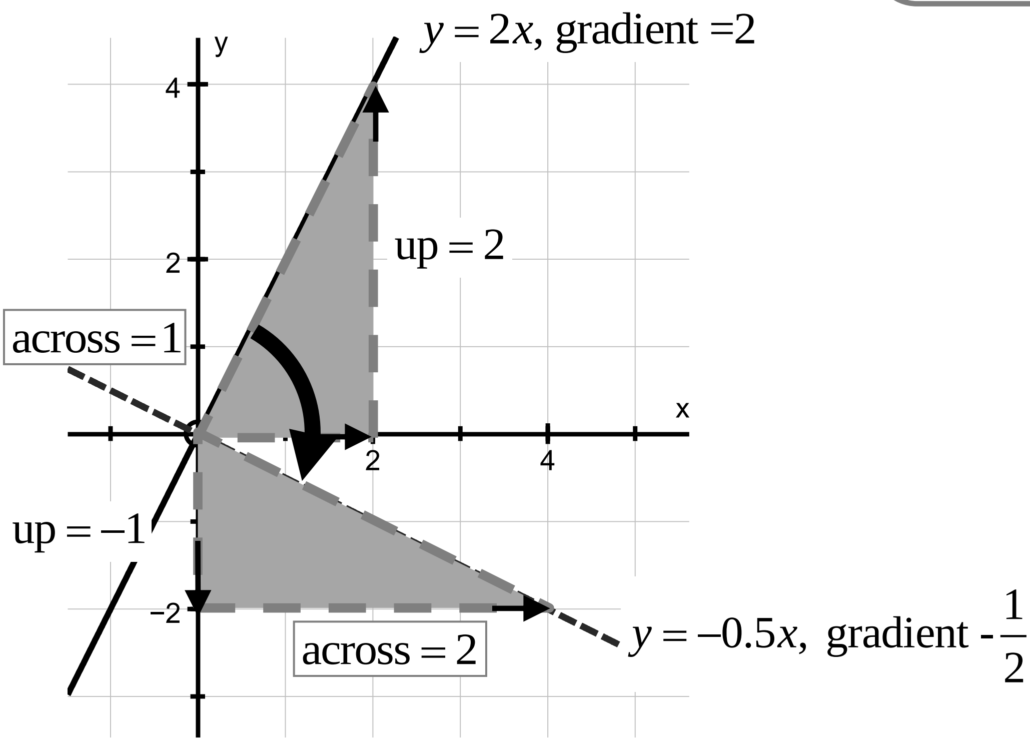


Pairs of perpendicular lines have gradients that multiply to make -1, for instance:

- y = x, y = -x (1 × (-1) = -1)
- y = 2x, y = -½x (2 × (-½) = -1)
- y = -10x+3, y = 0.1x+4 (-10 × -0.1 = -1).

The y-intercept values +3, +4 do not matter – they just slide the line up and down without changing its gradient.

Why?



When we rotate a line through 90°,

- the triangle rotates too
- the "across" and "up" values get <u>swapped</u> and one becomes <u>negative</u>

If the gradient of one line is m_1, the gradient of the perpendicular is $$m_2 = -\frac{1}{m_1}$$

- e.g. If the first gradient = 10, the second must be $-\frac{1}{10}$.
- If the first gradient = -3, the second must be $\frac{1}{3}$.
- If the first gradient = $\frac{2}{3}$, the second must be $-\frac{3}{2}$ (its reciprocal, with the sign changed).

<u>In general, you always have:</u>

- one positive, one negative gradient
- one steep, one shallow gradient.