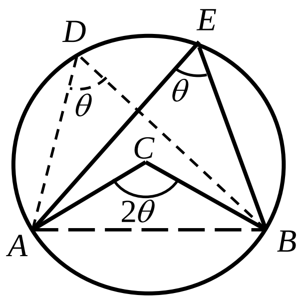
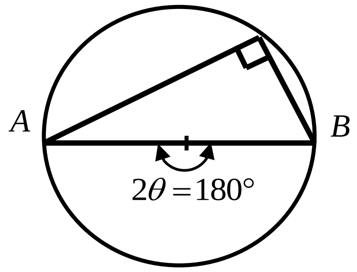
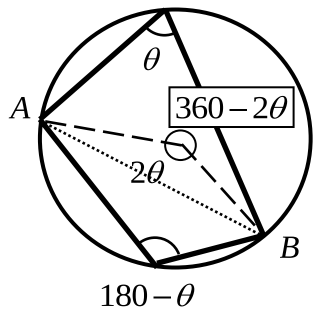
Angles in circles

(i) **Double angle rule**.

The angle at the centre is double the angle at the edge, for any two triangles sitting on the same side of a common "base chord" AB, i.e. "*in the same segment*".





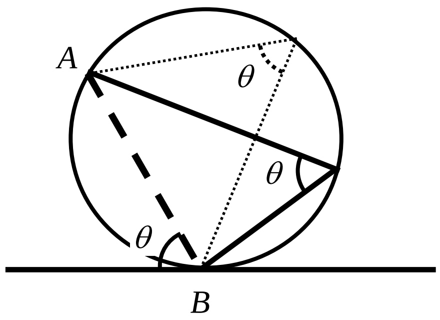


Hence:

- all triangles like ADB or AEB have the same angle θ on the circumference of the circle.
- if AB is a diameter, $2\theta = 180°$ so $\theta = 90°$
- Opposite angles in a cyclic quadrilateral add to 180° (because angles around the centre point add to 360°).

(ii) Alternate segment (tangent) rule

(Angle between the base chord AB and the tangent) =
(angle in opposite corner of any base chord triangle in the opposite segment) = θ

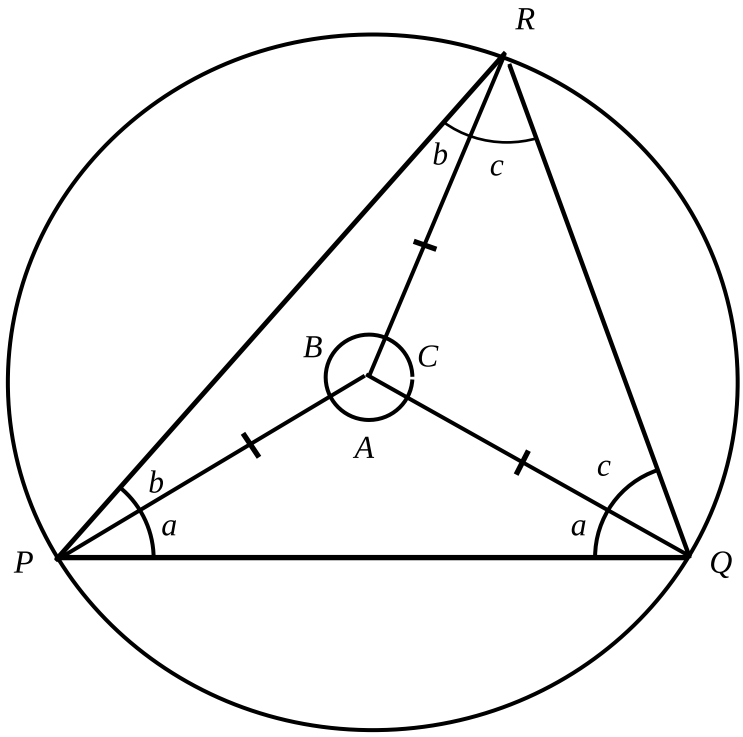


Circle proofs

(i) the double-angle rule

Your starting point should be to *assign a letter to each angle* (same letter if obviously equal, else use a different letter) and then *use algebra and angle rules* to find the relationship between the edge and centre angles.

In most double-angle questions you are looking for angles "*in the same segment*" (to one side of a base chord). The proof below does all 3 angles at once – those based on chord PQ, those from chord PR and those from chord QR.



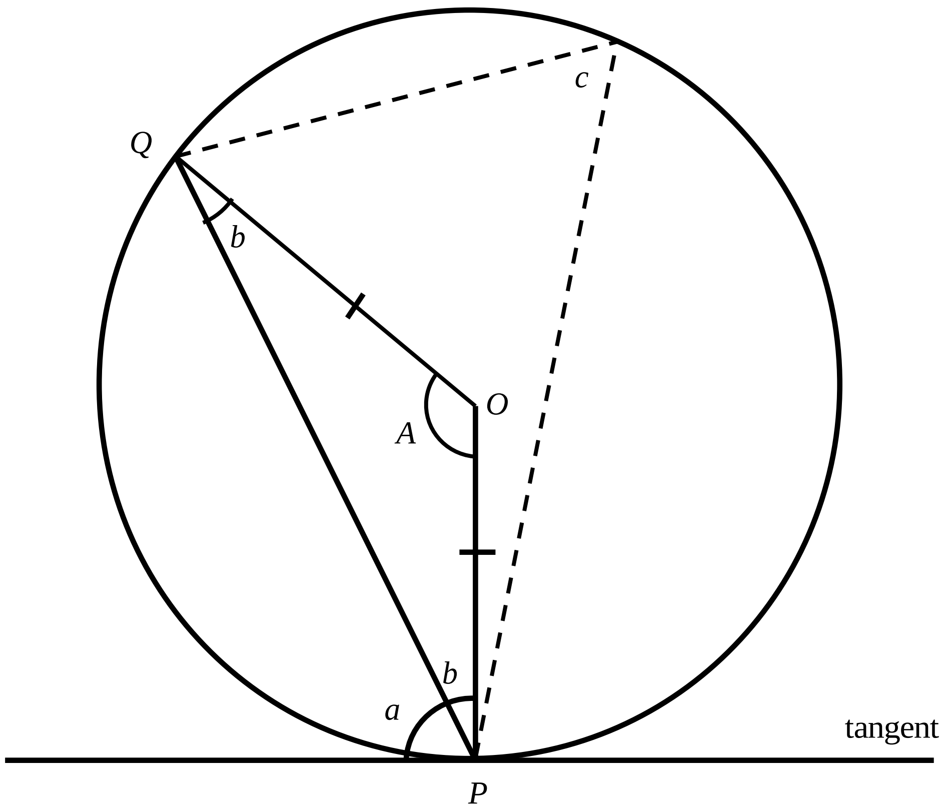
Each of the three triangles meeting at the centre is <u>isosceles</u> (same radius). Adding up the angles in the big triangle $2a+2b+2c=180°$ (angle sum in a triangle). We can halve this: $a+b+c=90°$, and *re-arrange it in various ways to get a formula for each of the angles at the three corners:*
$b+c=90-a$, $a+c=90-b$, $a+b=90-c$

Angles at the centre: $A+B+C=360°$
Angle sum in the small triangles: $A+2a=180°$, $B+2b=180°$, $C+2c=180°$.
Rearrange this to define the centre angles *A*, *B*, *C*:
$A=2(90-a)$, $B=2(90-b)$, $C=2(90-c)$.

Therefore: $A=2(b+c)$, $B=2(a+c)$, $C=2(a+b)$. Each angle at the centre is twice the angle to the opposite sector.

(ii) the alternate-sector (tangent) rule

As before, we label each angle. QP is our base chord, OP is a radius, R is any point on the edge of the circle *in the segment opposite angle **a***.



$a+b=90°$ (radius perpendicular to its tangent)
$A+2b=180°$ (triangle OPQ is isosceles since two sides are the same radius: its angles add to 180°).

Rearrange these: $a=90-b$
$A=180-2b=2(90-b)$

Hence $A=2a$

We already know from the double angle rule that $A=2c$
Hence $2a=2c$

Therefore $a=c$