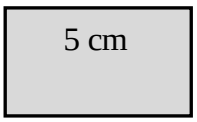
2-D shapes (polygons + circles).

Rectangle

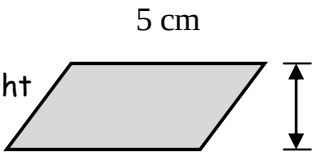
Area = width × height



$2\text{ cm} \times 5\text{ cm} = 10\text{ cm}^2$

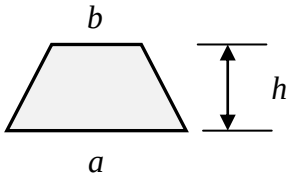
Parallelogram

Area = base width × vertical height

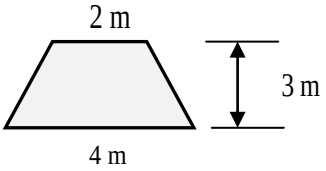


$2\text{ cm} \times 5\text{ cm} = 10\text{ cm}^2$

Trapezium



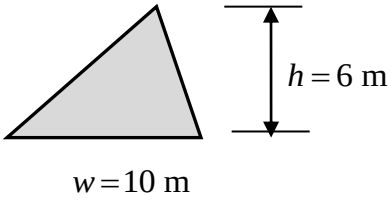
$$\text{Area} = \left(\frac{a+b}{2}\right)h$$



$$\text{Area} = \left(\frac{4+2}{2}\right)\times 3 = \frac{6}{2}\times 3 = 3\times 3 = 9\text{ m}^2$$

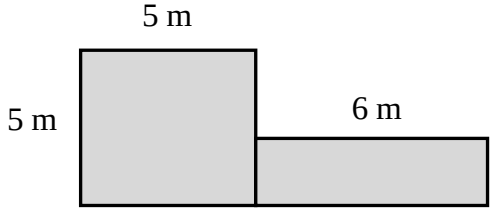
Triangle

Area = base width × vertical height ÷ **2**



$$A = \frac{wh}{2} = \frac{10\times 6}{2} = \frac{60}{2} = 30\text{ m}^2$$

Collections of shapes: "add the areas":

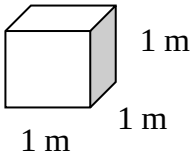


$$\text{Area} = 5\times 5 + 6\times 2 = 25 + 12 = 37\text{ m}^2$$

3D shapes.

Surface area: add up the area of each surface.

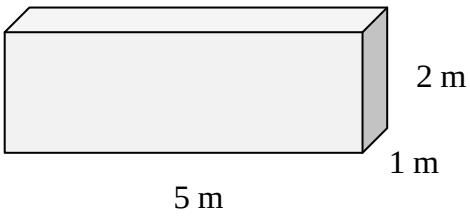
Cube with 1 m sides:



Each face has area 1m × 1m = 1m^2.

The cube has 6 faces (like a dice) so the total surface area = 6m^2.

Cuboid:



Add up all the areas:

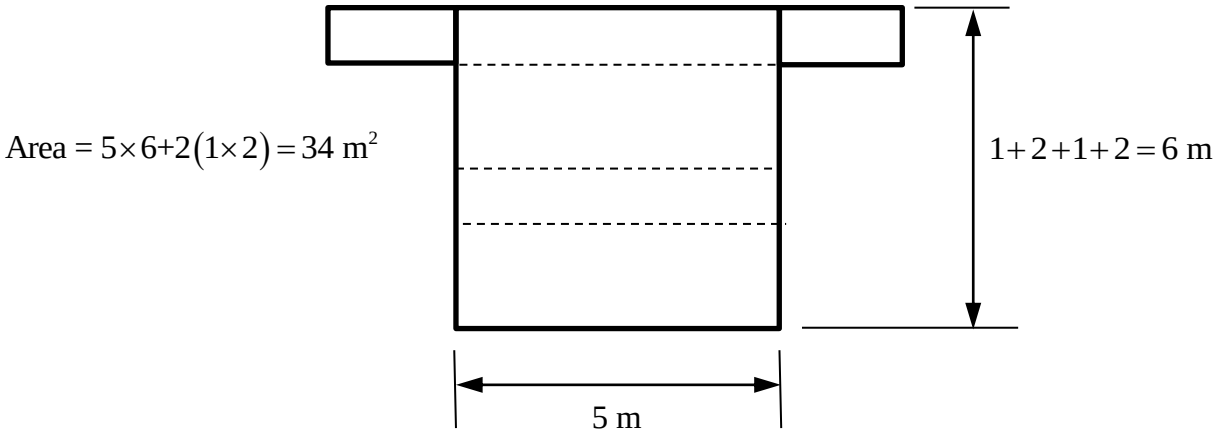
Front area = 5×2 = 10 m^2, front + back = 10+10 = 20 m^2.

Top area = 5×1 = 5 m^2, top + bottom = 5+5 = 10 m^2

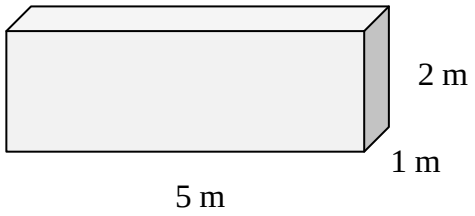
End area = 1×2 = 2 m^2, left end + right end = 2+2 = 4 m^2.

Total 20 + 10 + 4 = 34 m^2

or, unwrap it to make a net:



Volume of a cuboid.



Volume = width × depth × height

= 5 × 1 × 2 = 10 m^3.

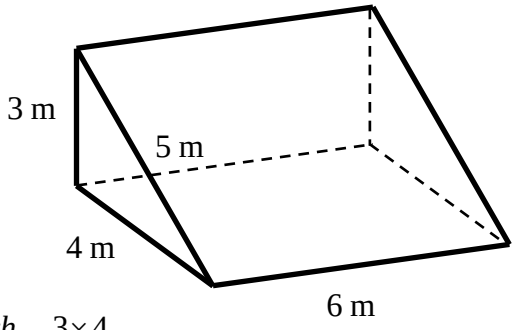
This is just a special case of the general rule:

Volume of a prism = (base area) × height

(A prism is anything with a constant cross-section, like a stick of Blackpool rock).

Examples

Find the surface area and volume of this triangular prism.



Cross-section area = $\frac{wh}{2} = \frac{3 \times 4}{2} = 6\ m^2$

Volume = A×L= 6 × 6 = 36 m^3.

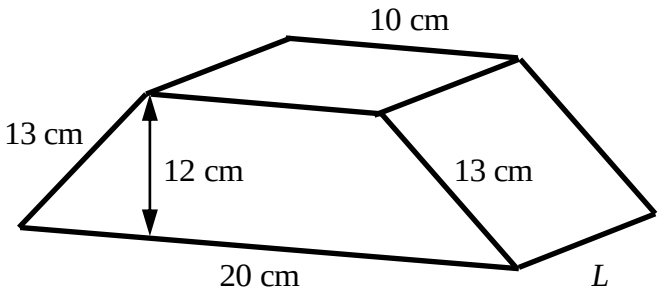
Surface area = 2×6 + (3+4+5) ×6 = 12 + 12×6 = 84 m^2.

Really awkward problems where a length must be found

- Decide which of the above formulae you need
- Write an equation in the form "formula = value"
- Solve the equation

Examples:

(a) The volume of this trapezoidal prism = 3780 cm^3. Find the surface area.



Area of the cross-section (the trapezium) = $\left(\frac{a+b}{2}\right)h = \left(\frac{10+20}{2}\right)\times 12 = 15\times 12 = 180 \text{ cm}^2$

We know the volume, so can find the length L:

$AL = 180L = 3780$ **(equation)**

$L = \frac{3780}{180} = 21 \text{ cm}$

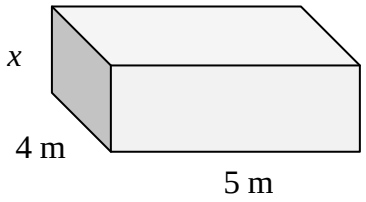
Now we find the surface area:

Surface area = $2\times 180 + (10+13+20+13)\times 21 = 360 + 1176 = 1536 \text{ cm}^2$

(= two trapeziums plus a ribbon of the four rectangular sides).

(b) The surface area of this cuboid = 112 cm^2. Find the volume.

First we must find the height x.



Surface area = $2(4\times5)+(4+5+4+5)x=40+18x$

Hence we need $40+18x=112$ **(equation)**

$18x=112-40=72$

$x=\frac{72}{18}=4\text{ m}$

nb. Each of these lines takes an expression and simplifies it.

> Please do not tell me that 40+18x = 112 – 40 = 72÷18 = 4, this is horribly wrong! If you do this you are abusing the equals sign. You cannot use = unless the values are the same on both sides.

Now we know x, the volume is 4×4×5 = 80 m^3.