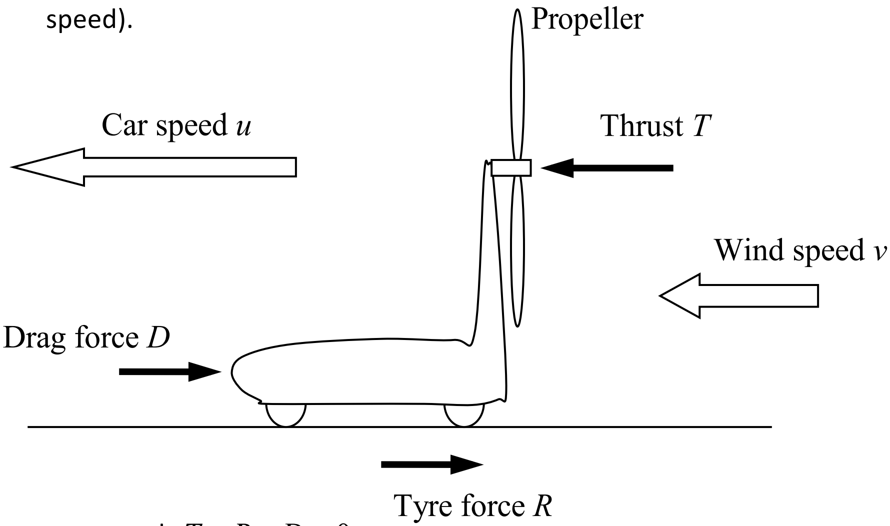
The "faster than wind" car (downwind)

The first question is: "What are we looking at?" Do we have a wind turbine that is geared to drive the wheels or wheels that are geared to drive a propeller?.

- If it were a wind turbine, it would make no power when the car speed = wind speed since the "apparent wind" speed would be zero. You would not be able to cross this "wind speed barrier".
- A propeller, though, can happily push against air whether there is a wind speed or not. We are transferring power (force × speed) from a power source moving at high speed (the ground) to do work on something moving at a relatively low speed (the apparent wind speed).



At constant speed, $T - R - D = 0$

Assume the propeller is sized to produce a thrust that is a multiple of the drag, $T = kD$

Define a propeller and transmission efficiency η such that:

$$(\text{propeller thrust})\times(\text{apparent wind speed}) = \eta(\text{shaft power input})$$

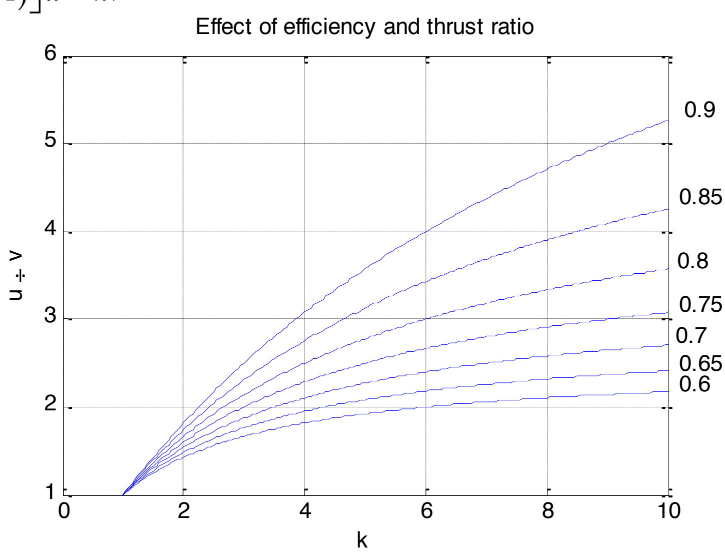
Then an energy balance gives $T(u-v) = \eta R u$

Now $T = kD$ and $R = T - D = (k-1)D$

Hence $kD(u-v) = \eta(k-1)Du$ and $\left[k - \eta(k-1)\right]u = kv$

$$\frac{u}{v} = \frac{k}{k(1-\eta)+\eta} = \frac{1}{(1-\eta)+\dfrac{\eta}{k}}$$

As one would expect, to get high $\frac{u}{v}$ one needs high η and k values.



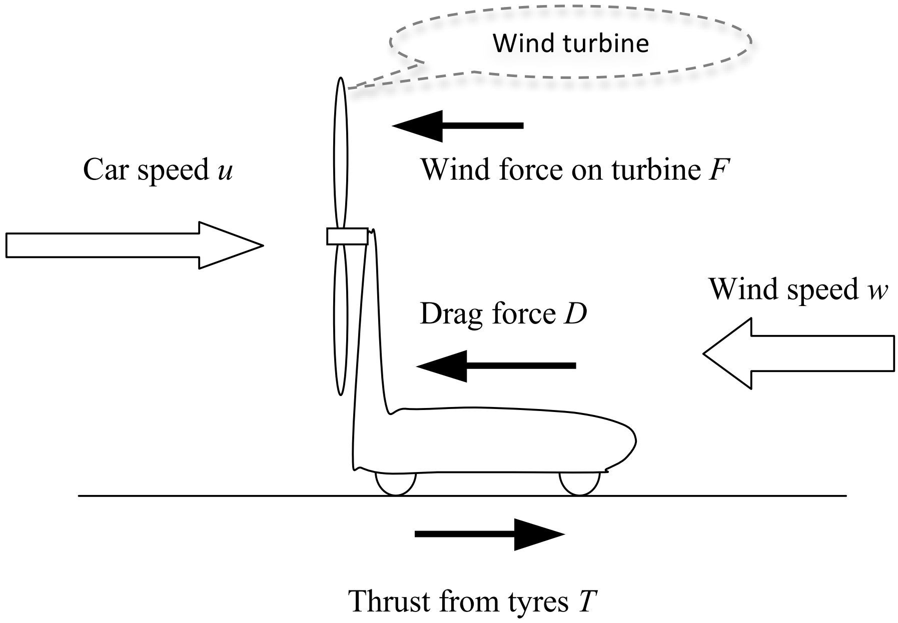
The "faster than wind" car (upwind)

To drive into the wind, we need the opposite arrangement. We are transferring power from a high-speed fluid (the wind, apparent speed $w+u$) to a slower "sink", the ground moving under the car at speed u.

Since power = force × speed, if there were no energy lost ("inefficiency") we could say

power in = power out

$F(w+u) = Tu$, $T = F\left(\frac{w+u}{u}\right)$ so $T > F$ and the car will accelerate into the wind.



Now let us account for drag and inefficiency. Let $F = kD$ and $Tu = \eta F(w+u)$. The car will asymptote towards a speed u such that $F + D = T$.

Then $\left(1+\frac{1}{k}\right)Fu = \eta F(w+u)$

$$\left(\frac{k+1}{k}\right)u = \eta w + \eta u$$

$$\left(\frac{k+1}{k} - \eta\right)u = \eta w$$

$$\left(\frac{k(1-\eta)+1}{k\eta}\right)u = w$$

$$\frac{u}{w} = \frac{k\eta}{k(1-\eta)+1}$$