M2 JUNE 2010

1. $a = (3t+5)\ m/s^2$

$v = \int a\,dt = \frac{3}{2}t^2 + 5t + c$

At $t=0$, $v = c = 2$ $\quad \therefore v = \frac{3}{2}t^2 + 5t + 2$

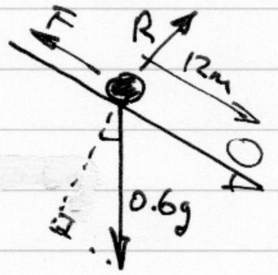
At $t=T$, $\frac{3}{2}T^2 + 5T + 2 = 6$

$3T^2 + 10T + (4-12) = 0 \qquad ac = -24 = 12\times -2$

$(3T - \frac{2}{1})(1T + \frac{12}{3}) = 0,$

$T = \frac{2}{3}$ (or -4 ×).

2.(a)



Increase in KE = loss of PE − work against friction.

$\frac{1}{2}\times 0.6\times 4^2 = 0.6\times 9.8(12\sin 30) - 12F$

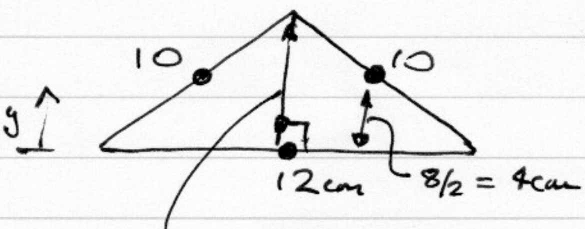
$\therefore$ Work against friction $= 12F = 30.48$ J

(b) $F = \frac{30.48}{12} = 2.54N$, Sliding $\therefore F = F_{max} = \mu R$

Resolve ↗, $R - 0.6g\cos 30 = 0$, $R = 0.6g\cos 30$

$\therefore \mu = \frac{F}{R} = \frac{2.54}{0.6\times 9.8\cos 30} = 0.499$

3(a)



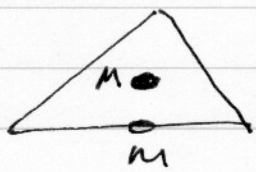
$\sqrt{10^2 - 6^2} = \sqrt{64} = 8$

mass ∝ length

$\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{2(10\times 4) + 12\times 0}{10+10+12} = \frac{80}{32}$

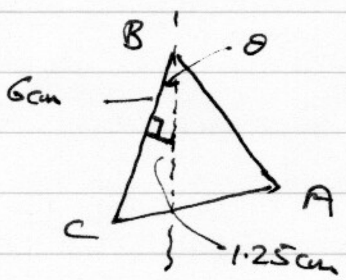
$\therefore \bar{y} = 2.5$ cm

(b) Equivalent system



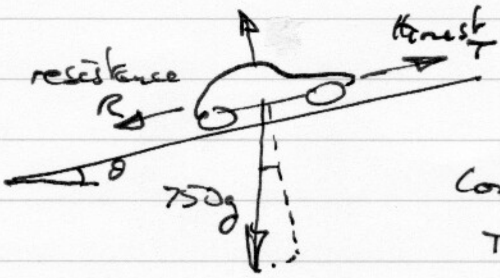
$\bar{y} = \frac{M\times 0 + M\times 2.5}{2M}$

$= 1.25$ cm



$\tan\theta = \frac{1.25}{6}$ $\qquad \therefore \theta = 11.8°$

4.(a)



Power = force × speed

$15000 = 20T, \quad T = 750\,N$

Constant speed $\therefore a = 0$

$T - R - 750g\sin\theta = 0$

$R = 750 - \frac{750 \times 9.8}{15} = 260\,N$

(b) 18000 watts $= 20T, \quad T = 900\,N$

$T - R - 750g\sin\theta = ma,$

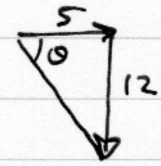
$900 - 260 - \frac{750 \times 9.8}{15} = 750a \quad \therefore a = 0.2\,m/s^2$

5(a) $\underline{u} = \begin{bmatrix}10\\24\end{bmatrix} m/s, \quad \underline{v} = \begin{bmatrix}20\\0\end{bmatrix} m/s$

$I = m\underline{v} - m\underline{u} = 0.5\left(\begin{bmatrix}20\\0\end{bmatrix} - \begin{bmatrix}10\\24\end{bmatrix}\right) = 0.5\begin{bmatrix}10\\-24\end{bmatrix} = \begin{bmatrix}5\\-12\end{bmatrix}$ Ns

Magnitude of impulse $= \sqrt{5^2+12^2} = 13$ Ns

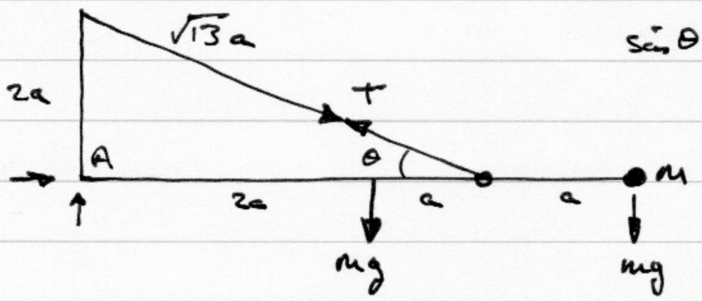
(b)



$\tan\theta = 12/5 = 2.4, \quad \theta = 67.4°$

(c) $KE_{lost} = KE_{initial} - KE_{final} = \frac{1}{2}m\left((10^2+24^2) - 20^2\right) = 69J$

6(a)



$\sin\theta = \frac{2a}{\sqrt{13}a} = \frac{2}{\sqrt{13}}$

Moments ↺ about A: $3aT\sin\theta - 2amg - 4amg = 0$

$\therefore T = \frac{6mg}{3\left(\frac{2}{\sqrt{13}}\right)} = mg\sqrt{13}$

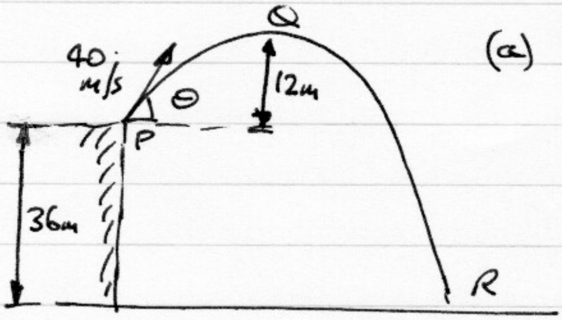
(b) Moments about A: $3aT\sin\theta - 2amg - 4aMg = 0,$

$T = \frac{2ag(m+2M)}{3a\,(2/\sqrt{13})} = \frac{\sqrt{13}}{3}g(m+2M).$

$T \leq 2mg\sqrt{13} \quad \therefore \quad \frac{\cancel{\sqrt{13}}}{3}\cancel{g}(m+2M) \leq 2m\cancel{g}\cancel{\sqrt{13}}, \quad m+2M \leq 6m$

$M \leq \frac{5}{2}m$

7.



(a) At Q, vertical velocity = 0

$v^2 = u^2 + 2as$

$0^2 = (40\sin\theta)^2 + 2(-9.8)12$

$\therefore 40\sin\theta = \sqrt{235.2} = 15.34$

$\sin\theta = 0.3834,$

$\theta = 22.54°$

(b) At R, $s = ut + \frac{1}{2}at^2$

$-36 = (40\sin\theta)t + \frac{1}{2}(-\ \ )t^2$

$4.9t^2 - 15.34t - 36 = 0$

$t = 4.695$ or -1.565 sec.

Then $OR = (40\cos\theta) \times 4.695 = 173.5\,m$

(c) $v^2 = u^2 + 2as = 40^2 + 2 \times 9.8 \times 36 = 2305.6,$

same direction

$v = 48.02\,m/s,$

$= 48.0\,m/s$ to 3 s.fig.

8. before | after $\longrightarrow$ +ve velocity.

(a) u $-u$ | V_A V_B

(A) (B) | (A) (B)

$3m$ m

Momentum : $3mu + m(-u) = 3mV_A + mV_B$

$\therefore 2u = 3V_A + V_B$

Restitution $e = \dfrac{V_B - V_A}{u-(-u)} = \frac{1}{2}$ $\therefore V_B - V_A = \frac{1}{2}(2u) = u$

Eliminate V_B : $(3V_A + V_B) - (V_B - V_A) = 4V_A = 2u - u = u$

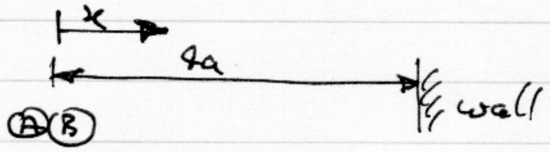
(i) $\therefore V_A = \dfrac{u}{4}$

(ii) $V_B = V_A + u = \dfrac{5}{4}u$

(b) $(V_B) \rightarrow$ then W_B $\dfrac{\text{sep. speed}}{\text{app. speed}} = \dfrac{-W_B}{V_B} = 2/5$

$\therefore W_B = -2/5(5/4\,u) = -\frac{1}{2}u.$

(c)



collision, $t=0$

Then $V_A = \frac{1}{4}u$, $V_B = \frac{5}{4}u$

It takes B $\frac{4a}{5/4u} = \frac{16a}{5u}$ seconds to reach the wall.

B bounces back with speed $\frac{1}{2}u$

$\therefore$ At time t $\left(t > \frac{16a}{5u}\right)$,

A is at $x = \frac{1}{4}ut$

B is at $x = 4a - \frac{1}{2}u\left(t - \frac{16a}{5u}\right)$

$\therefore$ They collide again $(t=T)$ when $\frac{1}{4}uT = 4a - \frac{1}{2}u\left(T - \frac{16a}{5u}\right)$

$= 4a - \frac{1}{2}uT + \frac{8}{5}a$

$\therefore$ $\frac{3}{4}uT = \frac{28}{5}a$

$T = \frac{28}{5}a \times \frac{4}{3u} = \frac{112a}{15u}$