MAY 2009 M2

1. $\underline{I} = m\underline{v} - m\underline{u}$

$\underline{v} - \underline{u} = \underline{I}/m, \quad \underline{v} = \underline{u} + \underline{I}/m$

$= \begin{pmatrix} 3 \\ 7 \end{pmatrix} + \dfrac{\begin{pmatrix} 5 \\ -3 \end{pmatrix}}{0.25} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 20 \\ -12 \end{pmatrix} = \begin{pmatrix} 23 \\ -5 \end{pmatrix}$ m/s

Speed $= \sqrt{23^2 + 5^2} = 23.54$ m/s

2 (a) $v = 8t - t^2, \quad dv/dt = 8 - 2t$

At max v, $dv/dt = 0 \therefore \; 2t = 8, \quad t = 4$ sec

$v = 8 \times 4 - 4^2 = 16$ m/s

(b) When it returns to O, $s = 0$.

$s = \int v\,dt = 4t^2 - \frac{1}{3}t^3 + c,$

$c = 0$ since starts at origin

$\therefore$ Need $4t^2 - \frac{1}{3}t^3 = 0$

$t^2(4 - \frac{1}{3}t) = 0, \quad t = 12$ sec.

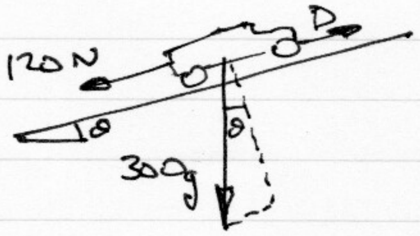
3. 

Constant speed,

$D - 120 = 0,$

$D = 120$ N.

(a) Power $= Dv = 120 \times 10 = 1200$ W

(b) 

Resolve ↗

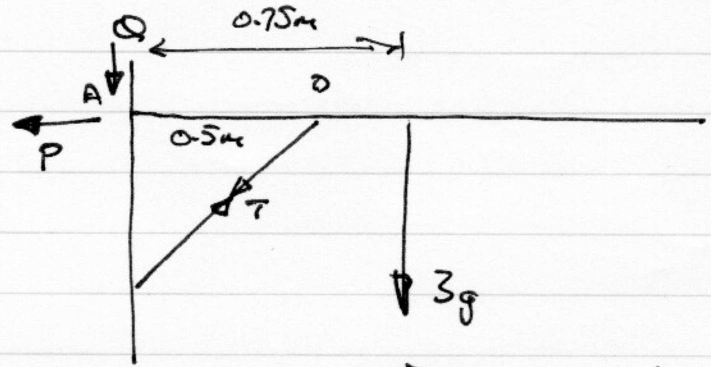
$D - 300g \sin\theta - 120 = 0,$

$D = 120 + \dfrac{300g}{14} = 330$ N

Power $= Dv$,

$1200 = 330\,v, \quad v = 3.64$ m/s

4 (a)



Moments about A: $0.5T\sin 45 - 3g \times 0.75 = 0$

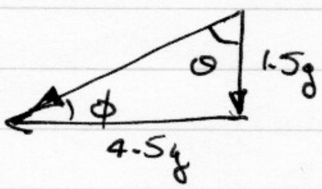
$$T = \frac{3g\,0.75}{0.5\sin 45} = 62.37\text{ N}$$

(b) Resolve ←

$P - T\cos 45 = 0, \quad P = T\cos 45 = 4.5g = 44.1\text{ N}$

Resolve ↓

$Q + 3g - T\sin 45 = 0, \quad Q = 4.5g - 3g = 1.5g = 14.7\text{ N}$



magnitude $\sqrt{44.1^2 + 14.7^2} = 46.5\text{ N}$

$\tan\theta = 3, \quad \theta = 71.6°$ to the vertical,

$\phi = 18.4°$ to the horizontal.

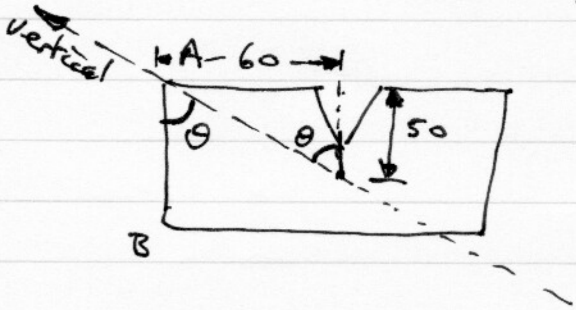
5(a) Define x down from AD ↓x

Rectangle: area $90 \times 120 = 10800\text{ cm}^2, \; \bar{x} = 45$

Triangle: area $\tfrac{1}{2}(60 \times 60) = 1800\text{ cm}^2, \; \bar{x} = \frac{0+0+60}{3} = 20$

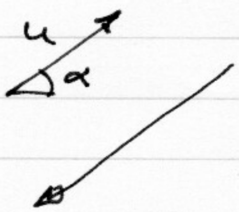
$$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{10800 \times 45 + (-1800) \times 20}{10800 - 1800} = 50\text{ cm}$$

(b)



$\tan\theta = \frac{60}{50} = 1.2,$

$\theta = 50.2°$

6(a) 

Horizontally: $(u\cos\alpha)t = 10$

Vertically: $(u\sin\alpha)t - \tfrac{1}{2}gt^2 = 2$

$$ut = \frac{10}{\cos\alpha}, \quad t = \frac{10}{u\cos\alpha}$$

$$(ut)\sin\alpha - \frac{g}{2}t^2 = 2$$

$$10\frac{\sin\alpha}{\cos\alpha} - \frac{g}{2}\,\frac{100}{u^2\cos^2\alpha} = 2$$

$$\therefore\ 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha} = 2$$

(b) $\alpha = 45^\circ \rightarrow \tan\alpha = 1,\ \cos^2\alpha = \frac{1}{2}$

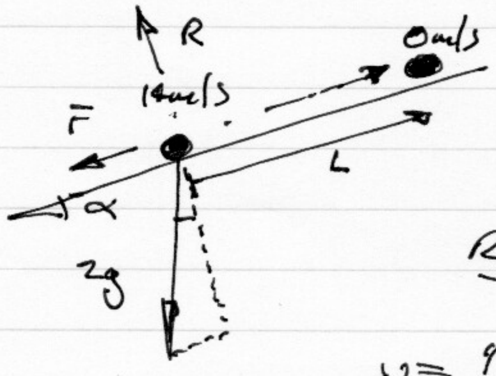
$$10 - \frac{100g}{u^2} = 2, \quad 8 = \frac{100g}{u^2}, \quad u^2 = \frac{100g}{8}$$

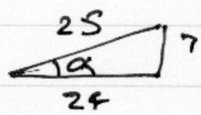
$v^2 = u^2 + 2as$ — a & s in opposite directions

$$= 12.5g + 2\times g\,(-2) = 8.5g$$

$\therefore\ v = 9.13$ m/s

7.(a) (Gain in PE) = (loss of KE) − (work against friction)





Resolve ↑ $R - 2g\cos\alpha = 0$

$$\therefore R = 2g\left(\tfrac{24}{25}\right) = \frac{48}{25}g$$

$\mu = 1/8$

Sliding $\therefore F = F_{max} = \mu R = \frac{6}{25}g$

work equation:

$$\underbrace{2gL\sin\alpha}_{\text{gain in PE}} = \underbrace{\tfrac{1}{2}\times 2\,(14^2 - 0^2)}_{\text{loss of KE}} - \underbrace{\left(\tfrac{6}{25}g\right)L}_{\text{friction loss.}}$$

$$gL\left(2\times\tfrac{7}{25} + \tfrac{6}{25}\right) = 196$$

$(9.8 \times {}^{20}/_{25}) L = 196$, $\therefore L = 25m$

(b) Increase in KE = loss of PE − work against friction

$\frac{1}{2}m(v^2 - 0^2) = 2g(25\sin\alpha) - \frac{6}{25}g(25)$.

$= 14g - 6g = 8g$

$\frac{1}{2}m = 1$ $\therefore$ $v^2 = 8 \times 9.8 = 78.4$, $v = 8.85\ m/s$

8.(a) Initial:

	A	B
	4m	3m
	u	−V

then

A	B
0m/s	kV m/s

$\longrightarrow$ pos. velocity

Momentum: $4mu - 3mV = 3m(kV)$ $\therefore 4u - 3V = 3kV$

Restitution: $e = \dfrac{kV}{u+V} = 3/4$ $\therefore 4kV = 3u + 3V$

Eliminate k:

(×4) → $16u - 12V = 12kV$

(×3) $9u + 9V = 12kV$ −

$7u - 21V = 0$, $u = 3V$

(b) $4u - 3V = 4(3V) - 3V = 9V = 3kV$ $\therefore k = 3$

(c) A is now at rest.

Now:

B	C
3m	m
kV = 3V	−2V

after 2nd collision:

B	C
w_B	w_c

Momentum: $3m(3V) + m(-2V) = 3mw_B + mw_c$

$\therefore 7V = 3w_B + w_c$

Restitution: $\dfrac{w_c - w_B}{3V - (-2V)} = e$

The biggest $w_c - w_B$ will be when the collision is perfectly elastic ($e = 1$). $\therefore w_c - w_B = 5V$

Subtracting: $(3w_B + w_c) - (w_c - w_B) = 4w_B = 7V - 5V = 2V$

$\therefore w_B = \frac{V}{2}$, positive, B does not move back towards A.