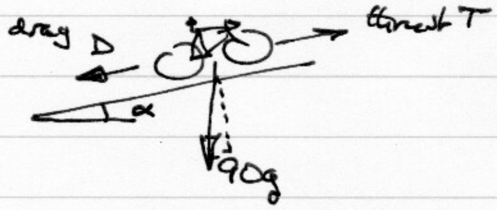
M2 JUNE 2007

1.



Power = force × speed

$444 = 6T, \quad T = 74\text{ N}$

Resolving ←: $D + 90g\sin\alpha - T = 0$

$D = 74 - 90 \times 9.8\left(\frac{1}{21}\right) = 32\text{ N}$

2. $m = 0.5\text{ kg}, \quad \underline{v} = 3t^2\underline{i} + (1-4t)\underline{j}$ m/s

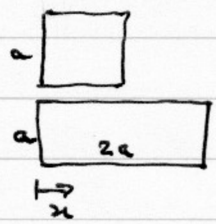
(a) $\underline{a} = \frac{dv}{dt} = (6t\underline{i} - 4\underline{j})$ m/s²

(b) $\underline{F} = m\underline{a} = 0.5(6t\underline{i} - 4\underline{j}) = (3t\underline{i} - 2\underline{j})$ N

At $t = 2$, $\underline{F} = (6\underline{i} - 2\underline{j})$ N

Magnitude $= |F| = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.325$ N

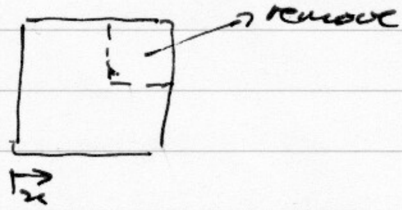
3.(a) Either add:



$\frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{(2a^2)a + (a^2)\frac{a}{2}}{3a^2} = \frac{2\frac{1}{2}a}{3}$

$\bar{x} = \frac{5}{6}a$

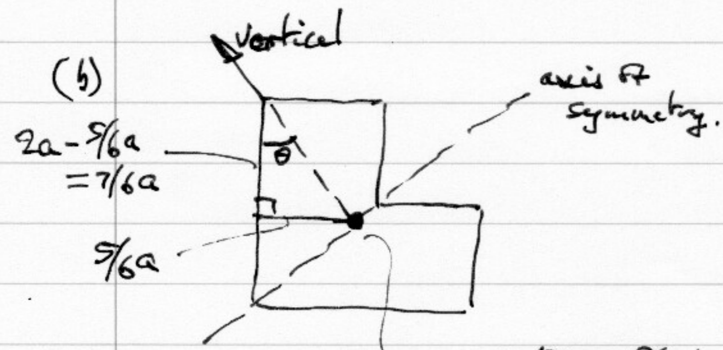
or subtract:



$\bar{x} = \frac{(4a^2)a - (a^2)\frac{3}{2}a}{3a^2} = \frac{2\frac{1}{2}a}{3}$

$= \frac{5}{6}a$

(b)

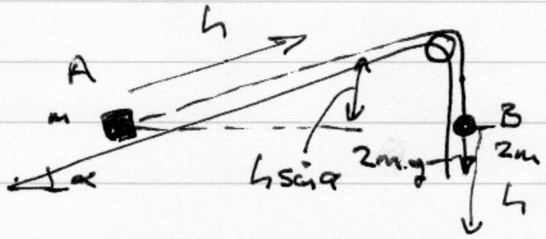


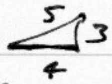
$\tan\theta = \frac{\frac{5}{6}a}{\frac{7}{6}a} = \frac{5}{7}$

$\therefore \theta = 35.54°$

$= 35.5°$ to 1 d.p.

4.



$\tan\alpha = 3/4$ 

$\therefore \sin\alpha = 3/5, \quad \cos\alpha = 4/5$

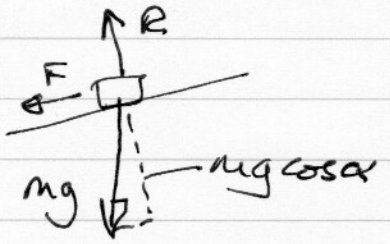
a) Loss of PE = loss by B − gain by A

$= 2mgh - mg(h\sin\alpha)$

$= (2 - 3/5)mgh = \frac{7}{5}mgh$

b) Start from rest, zero KE.

After moving h, gain of KE = loss of PE − work done against friction.



$R = mg\cos\alpha = 4/5\, mg$

sliding $\therefore F = F_{max} = \mu R$

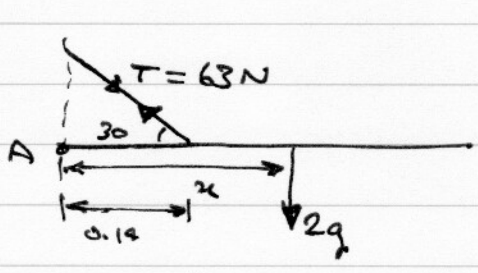
$= 5/8\,(4/5\, mg) = 1/2\, mg$

$\therefore KE = \frac{7}{5}mgh - (1/2\, mg)h$

$1/2\, mv^2 = \frac{9}{10}mgh$

$\therefore v^2 = \frac{2 \times 9gh}{10} = \frac{9}{5}gh$

5.(a)



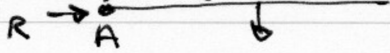
Moments about A ↻

$2gx - 63 \times 0.14 \sin 30 = 0$

$x = \frac{63 \times 0.14 \sin 30}{2 \times 9.8} = \frac{9}{40}$ = half of AB

$\therefore$ Length AB $= 2 \times \frac{9}{40} = \frac{9}{20} = 0.45$m

(b)



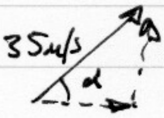
Resolve ↓ $F + 2g - 63\sin 30 = 0$

$\therefore F = \frac{63}{2} - 2 \times 9.8 = 11.9$N

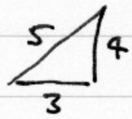
Resolve → $R - F\cos 30 = 0 \quad \therefore R = 63\cos 30 = \frac{63\sqrt{3}}{2}$ N

Magnitude of reaction $= \sqrt{R^2 + F^2} = 55.84$ N

6. (a)



$35\sin\alpha = 35\left(\frac{4}{5}\right) = 28$ m/s upwards.



$\tan\alpha = 4/3$
$\sin\alpha = 4/5$
$\cos\alpha = 3/5$

$v^2 = u^2 + 2as$, at max height $v = 0$

$\therefore 0^2 = 28^2 + 2(-9.8)s, \quad s = \frac{28^2}{2\times 9.8} = 40$ m above A.

(b) $35\cos\alpha = 35\left(\frac{3}{5}\right) = 21$ m/s horizontally.

Time to B: $t = \frac{168}{21} = 8$ sec

Vertical motion:

$$s = u_y t + \tfrac{1}{2}at^2 = 28t + \tfrac{1}{2}(-9.8)t^2 = -89.6\text{m},$$

A is 89.6 m above the ground.

(c) $v^2 = u^2 + 2as = 35^2 + 2(9.8 \times 89.6) = 2981.16,$

$v = 54.6$ m/s

$\uparrow \quad \uparrow$ a and s in same direction.

$\xrightarrow{+}$ velocity

7 (a) (i) ~~before~~:

u	0
P	Q
m	$5m$

after:

v_p	v_q
P	Q

Restitution:

$$e = \frac{\text{sep. speed}}{\text{app. speed}} = \frac{v_q - v_p}{u}$$

Momentum $mu = mv_p + 5mv_q$

$\rightarrow u = v_p + 5v_q$

$\therefore v_q - v_p = eu$

$5(v_p - v_q) = -5eu$

$$(v_p + \cancel{5v_q}) + (5v_p - \cancel{5v_q}) = u + -5eu$$

$$\therefore 6v_p = (1-5e)u, \qquad v_p = \frac{u}{6}(1-5e)$$

(ii) $v_q - v_p = eu, \quad v_q = v_p + eu = u\left(\frac{1}{6} - \frac{5}{6}e + e\right) = \frac{u}{6}(1+e)$

$$= \left(\frac{1+e}{6}\right)u$$

positive velocity.

7(b) $\quad 0 \quad\quad u \quad\quad 0$

(A) $\quad$ (B) $\quad$ (C)

$5m \quad\quad m \quad\quad 5m$

B and C equivalent to P and Q in part (a).

$\therefore$ After collision, B has velocity $\frac{u}{6}(1-5e)$.

$e = 4/5$ so $v_B = \frac{u}{6}(1-4) = \frac{-3}{6}u = -\frac{1}{2}u$

v_B is negative so it moves to left & hits A.

(c) After B hits A, it bounces back with half its former speed, $v_{B2} = -\frac{1}{2}(v_B) = \frac{1}{4}u$

After the first collision, $v_c = v_q = \left(\frac{1+e}{6}\right)u = \frac{1.8}{6}u = 0.3u$

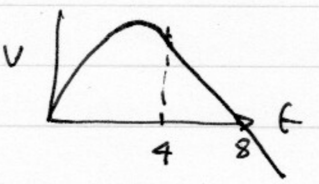
$\therefore v_c > v_{B2}$, B does not catch up, no further collision.

8 (a) $0 \le t \le 4$, $v = 8t - \frac{3}{2}t^2$ $\quad dv/dt = 8 - 3t$, $\quad dv/dt = 0$ at $t = 8/3$

Max $v = 8(8/3) - \frac{3}{2}(8/3)^2 = \frac{32}{3} = 10.66$ m/s

(b) $s = \int v\,dt = 4t^2 - \frac{1}{2}t^3 + c$ $\quad$ At origin at $t=0$ $\therefore c = 0$

At $t = 4$, $s = 4(4^2) - \frac{1}{2}(4^3) = 32$ m

(c) 

$v = 0 \therefore 16 - 2t = 0$, $t = 8$ sec.

(d) For $t > 4$, $s = 32 + \int_4^t 16 - 2t\,dt = 32 + \left[16t - t^2\right]_4^t$

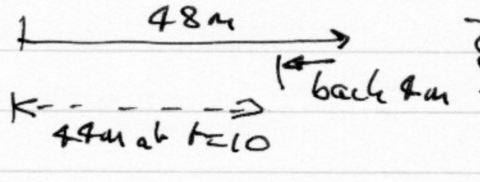
At $t = 10$,

$s = 32 + \left[16t - t^2\right]_4^{10} = 32 + (160 - 100) - (64 - 16) = 44$ m

BUT for $8 < t < 10$ it has negative velocity

At $t = 8$ (furthest point):

$s = 32 + \left[16t - t^2\right]_4^8 = 32 + (128 - 64) - (64 - 16) = 48$ m

So it moves 

total 52m