M2 JUNE 2006

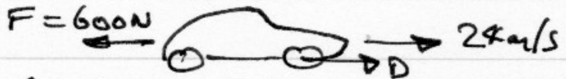
1. $a = (5-2t)\ m/s^2$

$v = \int a\,dt = \int 5-2t\,dt = 5t - t^2 + c$

At $t=0$, $v=6$ $\therefore c = 6$

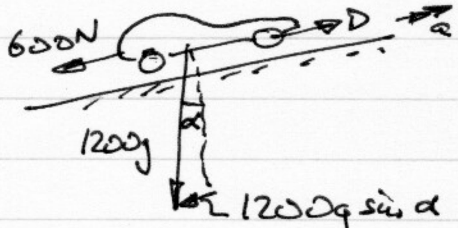
When P is at rest, $v=0$ $\therefore -t^2+5t+6=0$

$t^2-5t-6=0 = (t-6)(t+1)$ $\therefore (t>0)$ $\underline{t = 6\text{ sec}}$.

2. 

a) Constant speed $\therefore$ $D - F = 0$, driving force $D = 600N$.

Power = force × speed $= 600 \times 24 = 14400\ W = 14.4\ kW$

b) 

Resolving ↗ :

$D - 600 - 1200g\sin\alpha = 1200a$

At 20 m/s, $20D = 30000\ W$, $D = \frac{30000}{20} = 1500\ N$

$\therefore a = \frac{1500 - 600 - 1200 \times 9.8/28}{1200} = 0.4\ m/s^2$

3(a) $\underline{u} = -30i\ m/s$, $\underline{v} = (16i + 20j)\ m/s$.

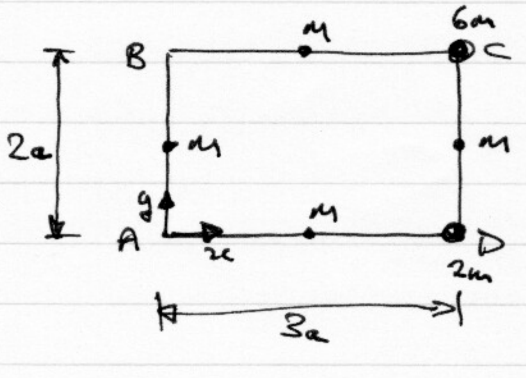
$I = m\underline{v} - m\underline{u} = 0.5\left(\begin{pmatrix}16\\20\end{pmatrix} - \begin{pmatrix}-30\\0\end{pmatrix}\right) = 0.5\begin{pmatrix}46\\20\end{pmatrix} = (23i + 10j)\ Ns$

Magnitude of impulse $= \sqrt{23^2+10^2} = 25.08\ Ns$

(b) $\underline{r} = [16t\underline{i} + (20t - 5t^2)\underline{j}]\ m$

$\underline{v} = \frac{dr}{dt} = (16\underline{i} + (20-10t)\underline{j})\ m/s$

At $t=3$, $\underline{v} = (16\underline{i} - 10\underline{j})\ m/s$, speed $= \sqrt{16^2+10^2} = 18.87\ m/s$

4. 

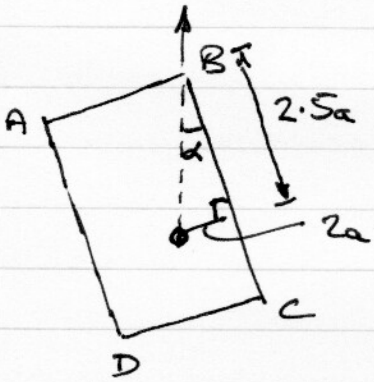
a)(i) $\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

$= \frac{0m + 2m(\frac{3}{2}a) + 9m(3a)}{4m + 6m + 2m} = \frac{30ma}{12m}$

$= 2.5a$ from AB

(ii) $\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{2m(0) + 2m(a) + 7m(2a)}{12m} = \frac{16ma}{12m} = \frac{4}{3}a$ from AD

(b)

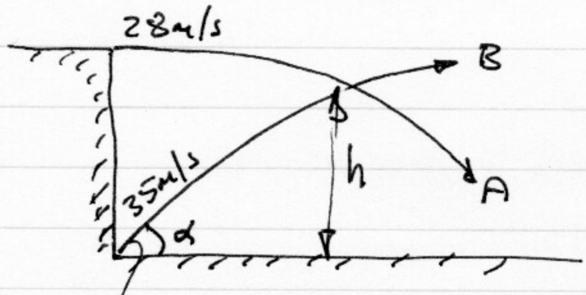


$2a - \frac{4}{3}a = \frac{2}{3}a$

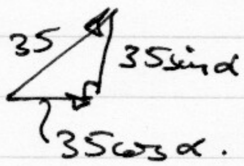
$\tan\alpha = \frac{\frac{2}{3}a}{2.5a} = \frac{4}{15}$

$\alpha = \tan^{-1}\left(\frac{4}{15}\right) = 14.93^\circ$

5(a)



Components 35



Stones collide at time t.
Distance travelled horizontally:

$28t = 35\cos\alpha\, t$

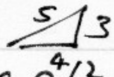
$\therefore \cos\alpha = \frac{28t}{35t} = \frac{4}{5}$

(b) At t, both stones are same height above ground.

For A, $h = 73.5 - \frac{1}{2}(9.8)t^2$

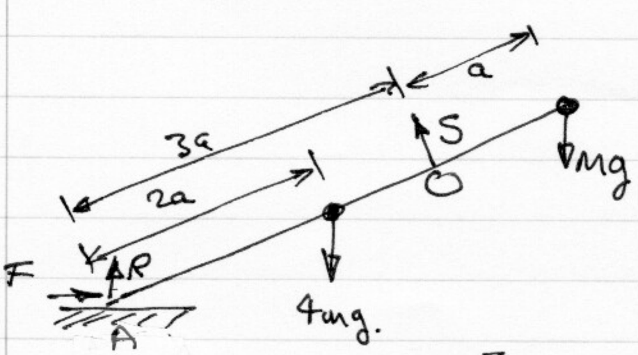
For B, $h = 35\sin\alpha\, t - \frac{1}{2}(9.8)t^2$

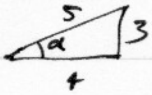
$\sin\alpha = \sqrt{1-\cos^2\alpha} = \frac{3}{5}$



$\therefore 73.5 - 4.9t^2 = 35\left(\frac{3}{5}\right)t - 4.9t^2$, $t = \frac{73.5}{35 \times 0.6} = 3.5$ sec

6.





(a) <u>Moments about A</u> ↻

$3aS - 4mg(2a\cos\alpha) - mg(4a\cos\alpha) = 0$

$3S = 12mg\left(\frac{4}{5}\right)$, $S = \frac{16}{5}mg$

(b) <u>Resolving vertically</u>

$R + S\cos\alpha - 4mg - mg = 0$ (since in equilibrium).

$R = 5mg - \frac{16}{5}mg\left(\frac{4}{5}\right) = \frac{61}{25}mg.$

<u>Resolve →</u>

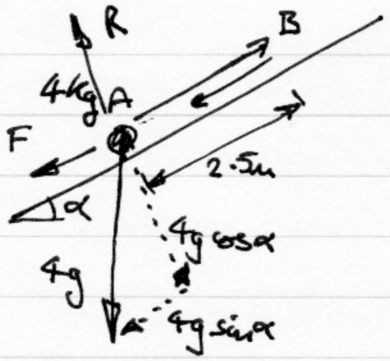
$F - S\sin\alpha = 0, \quad F = \frac{16}{5}mg\left(\frac{3}{5}\right) = \frac{48}{25}mg$

$F \leq \mu R \quad \therefore \frac{48}{25}mg \leq \mu\left(\frac{61}{25}mg\right), \quad \mu \geq \frac{48}{61}$

(c) S acts perpendicular to the plank because there is no friction here. (With friction S could include a sideways component

).

7.(a)

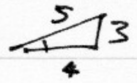


Moving up (A→B), friction acts downwards.

<u>Resolve ↑</u> $R - 4g\cos\alpha = 0$

$R = 4g\left(\frac{4}{5}\right) = \frac{16}{5}g.$

$F = \mu R = \frac{2}{7}\left(\frac{16}{5}\right) \times 9.8 = 8.96\ N$



Work done against friction

$= 8.96N \times 2.5m = 22.4\ J$

loss P.E.

(b) $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - 22.4 - 2.5\sin\alpha(4g) = 0$ (finishes at rest)

$\therefore \frac{1}{2}mu^2 = 81.2, \quad u^2 = \frac{2 \times 81.2}{4} = 40.6, \quad u = 6.37\ m/s$

(c) Moving down, R unchanged, F = 8.96N still but acts ↗.

$4g\sin\alpha - 8.96 = ma$

$a = \frac{14.56}{4} = 3.64\ m/s^2$ ↙.

$v^2 = u^2 + 2as = 0 + 2 \times 3.64 \times 2.5 = 18.2,$

$v = 4.27\ m/s$

8 (a) Before | After → positive velocity

Before		After	
(A)	(B)	(A)	(B)
m	$4m$	v_A	v_B
u	0 m/s		

Momentum : $mu + 4m(0) = mv_A + 4mv_B$, $\quad v_A + 4v_B = u$ — (1)

Restitution : $\dfrac{\text{separation speed}}{\text{approach speed}} = \dfrac{v_B - v_A}{u} = e$

$\therefore\ v_B - v_A = ue$ — (2)

Adding : (1)+(2) : $(v_A + 4v_B) + (v_B - v_A) = u(1+e)$

$5v_B = (1+e)u, \quad v_B = \dfrac{(1+e)}{5}u$

Then $v_A = u - 4v_B$

$= \left(1 - \dfrac{4(1+e)}{5}\right)u = \left(\dfrac{1-4e}{5}\right)u$

(b) "Direction of A is reversed" $\therefore v_A$ is negative, $1-4e<0$, $e > \frac{1}{4}$.

(B), v_B before impact, w_B after

$\dfrac{\text{sep. speed}}{\text{app. speed}} = \dfrac{-w_B}{v_B} = 4/5$,

$\therefore\ w_B = -4/5\, v_B = -\dfrac{4(1+e)}{25}u$

If B catches up with A & hits it a second time,

$\dfrac{4(1+e)u}{25} > |v_A|$, $\quad \dfrac{4(1+e)u}{25} > \dfrac{4e-1}{5}u$

$\therefore\ 4(1+e) > 5(4e-1)$, $\quad 4+4e > 20e-5$,

$9 > 16e$, $\quad e < \frac{9}{16}$

$\therefore\ \frac{1}{4} < e < \frac{9}{16}$

(c) With $e = 1/2$, $v_A = \left(\dfrac{1-4e}{5}\right)u = \dfrac{(1-2)}{5}u = -\dfrac{1}{5}u$

$v_B = \left(\dfrac{1+e}{5}\right)u = \dfrac{1\frac{1}{2}}{5}u = \dfrac{3}{10}u$

Energy loss $= \left(\frac{1}{2}mu^2\right) - \left(\frac{1}{2}m\left(-\frac{1}{5}u\right)^2 + \frac{1}{2}(4m)\left(\frac{3}{10}u\right)^2\right)$

$= \frac{1}{2}mu^2\left(1 - \left(\frac{1}{25} + \frac{36}{100}\right)\right) = \dfrac{3}{10}mu^2$