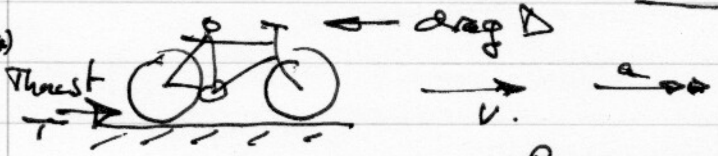
M2 Jan 2011

1(a)



Power = force × speed

Constant speed ⇒ $a = 0$, $T - D = 0$

$\therefore T = D = 32N$ $\qquad Tv = 384W$, $v = \frac{384}{32} = 12$ m/s

(b) At 9m/s, $T = \frac{384}{9} N$

$T - D = ma$, $\quad a = \dfrac{\frac{384}{9} - 32}{120} = \frac{4}{45}$ m/s²

2. $mv = mu + I$, $v = u + \frac{I}{m} = \begin{bmatrix}5\\1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}-6\\8\end{bmatrix} = \begin{bmatrix}2\\5\end{bmatrix}$ m/s

$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times (2^2 + 5^2) = 29 J$

3. $a = 4t^3 - 12t$

(a) $v = \int a\,dt = \int 4t^3 - 12t\,dt = t^4 - 6t^2 + c$

At $t = 0$, $v = 8$ $\therefore c = 8$, $\quad v = t^4 - 6t^2 + 8$.

(b) $s = \int v\,dt = \int t^4 - 6t^2 + 8\,dt = t^5/5 - 2t^3 + 8t + k$

$s = 0$ at $t = 0$ ⇒ $k = 0$ (or use definite integral).

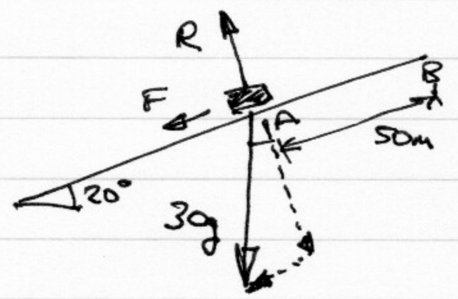
$\therefore s = t^5/5 - 2t^3 + 8t$

(c) If $v = 0$, $\quad t^4 - 6t^2 + 8 = 0$

$(t^2 - 4)(t^2 - 2) = 0$

$\therefore t^2 = 4$, $\underline{t = \pm 2}$ or $t^2 = 2$, $\underline{t = \pm\sqrt{2}}$ seconds.

4(a)



"Moving up":

$\underline{\text{Resolve}} \nwarrow$, $R - 30g\cos 20 = 0$

$\therefore R = 30g\cos 20$,

$F = \mu R = \frac{30}{4} g\cos 20$

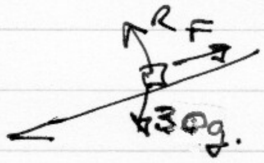
Work done $= \left(\frac{30}{4} g\cos 20\right) \times 50 + 30g(50\sin 20)$

$= 1500g\left(\frac{\cos 20}{4} + \sin 20\right) = 8481 J$

(b) Either $(KE+PE)_{final} = (KE+PE)_{initial}$ − work done against friction

$KE_A = 30g(50\sin 20) - 50F$

loss of PE

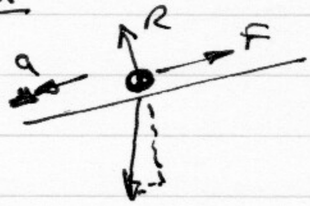


$= 1500g\left(\sin 20 - \frac{1}{4}\cos 20\right)$

$= 1574J$

$= \frac{1}{2}mv^2, \therefore v = 10.24$ m/s

Or



$F = \frac{30}{4}g\cos 20$ as before

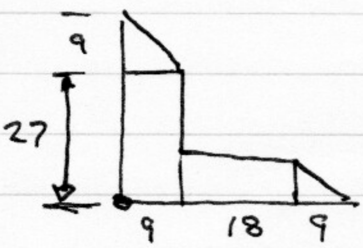
Resolve ↙ $30g\sin 20 - F = ma = 30a$

$\therefore a = g\sin 20 - \frac{1}{4}g\cos 20$

$v^2 = u^2 + 2as$

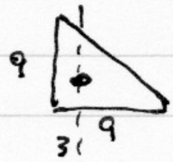
$= 0^2 + 2g(\sin 20 - \frac{1}{4}\cos 20)\times 50, \quad v = 10.24$ m/s

5(a)



Taking (0,0) as B

For a triangular lamina



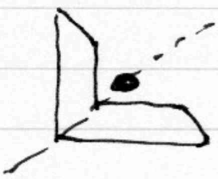
$\bar{x} = \frac{0+0+9}{3} = 3$

$\therefore \bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{\frac{1}{2}(9\times 9)\times 3 + (27\times 9\times \frac{9}{2}) + (18\times 9)\times 18 + \frac{1}{2}(9\times 9)\times 30}{2\times \frac{1}{2}(9\times 9) + 27\times 9 + 18\times 9}$

$= \frac{243/2 + \frac{2187}{2} + 2916 + 2430}{486} = 11$ cm. [or big − small].

→ see last page.

(b) By symmetry.



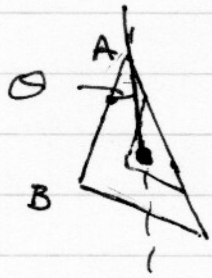
, distance from BC

= distance from AB

= 11 cm

so that centre of mass lies on the axis of symmetry

(c)

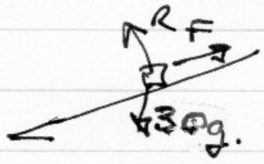


$\tan\theta = \frac{11}{25} \quad \therefore \theta \simeq 23.75°$

$\approx 24°$.

(b) Either $(KE+PE)_{final} = (KE+PE)_{initial}$ − work done against friction

$KE_A = \underbrace{30g(50\sin 20)}_{\text{loss of PE}} - 50F$



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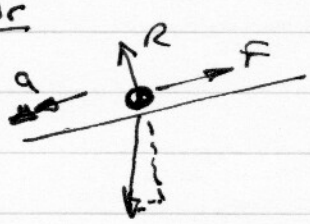

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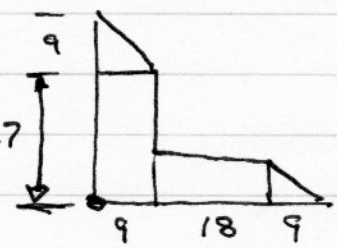

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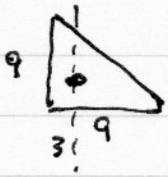

$\bar{x} = \frac{0+0+9}{3} = 3$

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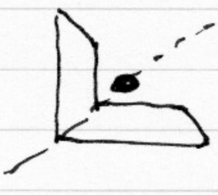

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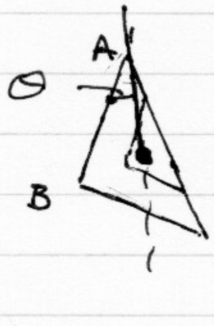

$\tan\theta = \frac{11}{25} \quad \therefore \theta = 23.75°$

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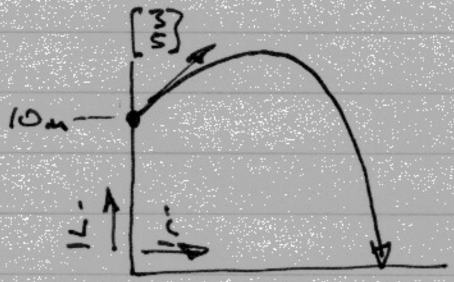
6.



$$\underline{a} = \begin{bmatrix} 0 \\ -9.8 \end{bmatrix} \text{ m/s}^2$$

(a) $\underline{s} = \begin{bmatrix} 0 \\ 10 \end{bmatrix} + \underline{u}t + \frac{1}{2}\underline{a}t^2$

$= \begin{bmatrix} 0 \\ 10 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} t + \begin{bmatrix} 0 \\ -4.9 \end{bmatrix} t^2$

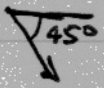
$= 3t\underline{i} + [10 + 5t - 4.9t^2]\underline{j}$ m.

(b) when hits ground, $10 + 5t - 4.9t^2 = 0$

$4.9t^2 - 5t - 10 = 0$

$t = \frac{5 \pm \sqrt{25+196}}{2 \times 4.9} = \frac{5 \pm \sqrt{221}}{9.8} = 2.027$ sec (if > 0).

(c) $\underline{v} = \underline{u} + \underline{a}t = \begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 \\ -9.8 \end{bmatrix} t = 3\underline{i} + (5 - 9.8t)\underline{j}$ m/s

(d) 

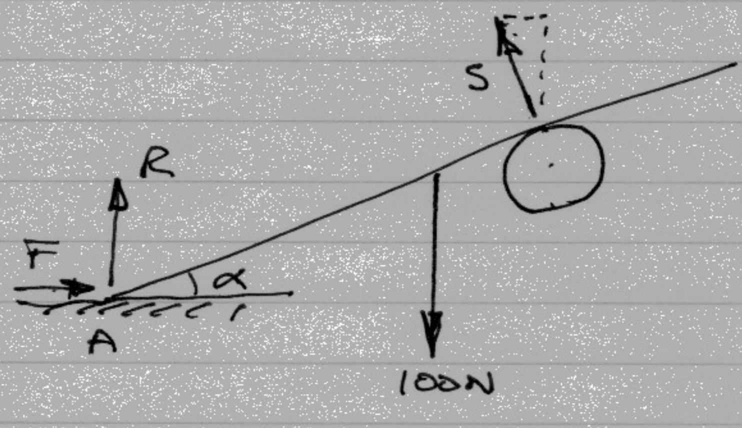
so $\underline{v} = k(\underline{i} - \underline{j})$

$\frac{5 - 9.8t}{3} = -1$, $\quad 5 - 9.8t = -3$, $\quad t = \frac{8}{9.8} = 0.816$ s.

(e) $\underline{v} = 3\underline{i} + (5 - 9.8 \times 8/9.8)\underline{j} = (3\underline{i} - 3\underline{j})$ m/s

Speed $= |v| = \sqrt{3^2 + 3^2} = 4.24$ m/s

7.



$\sin\alpha = \frac{1}{3}$

$\cos\alpha = \sqrt{1 - (1/3)^2} = \frac{\sqrt{8}}{3}$

Moments ↺ about A: $\quad 3S - 100(2\cos\alpha) = 0$

$S = \frac{200(\sqrt{8}/3)}{3} = 62.85$ N.

Resolve ↑ $\quad R + S\cos\alpha - 100 = 0$

$R = 100 - \frac{200\sqrt{8}}{9}\left(\frac{\sqrt{8}}{3}\right) = 100 - \frac{1600}{27} = \frac{1100}{27}$ N

Resolve $\rightarrow$ $\quad F - S\sin\alpha = 0$

$$F = \frac{200\sqrt{8}}{9}\left(\tfrac{1}{3}\right) = \frac{200\sqrt{8}}{27}$$

Equilibrium, $F \le \mu R$, $\quad \mu \ge \frac{F}{R}$

$\therefore \mu \ge \dfrac{\left(\frac{200\sqrt{8}}{27}\right)}{\left(\frac{1100}{27}\right)}$, $\quad \mu \ge \frac{2\sqrt{8}}{11}$,

$\therefore \mu \geq 0.514$, minimum $\underline{0.514}$.

8(a) Initial speed $u = 6\,m/s$, final speed v.

$e = \frac{v}{6} = \frac{1}{3} \quad \therefore v = 2\,m/s$

$\frac{1}{2}m(u^2 - v^2) = 64\,J \quad \therefore \quad m = \frac{2\times 64}{6^2 - 2^2} = 4\,kg.$

(b) 

Momentum: $2\times 3 + 4(-2) = 2v_q + 4v_p$

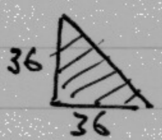
Restitution $\quad e = \dfrac{v_p - v_q}{3-(-2)} = \frac{1}{3} \quad \therefore \quad v_p - v_q = \frac{5}{3}$

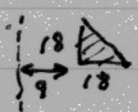
$(\times 2), \quad 2v_p - 2v_q = \frac{10}{3}$

Add: $\quad 6v_p = 6 - 8 + \frac{10}{3} = \frac{4}{3}$

$\therefore v_p = \frac{4}{18} = \frac{2}{9}\,m/s$, positive, moving $\rightarrow$

so hits wall again.

5(a) by subtraction 

$\bar{x} = \frac{36}{3} = 12$


$\bar{x} = \frac{18}{3} = 6+9 = 15$

$$\text{Overall, } \bar{x} = \frac{\left(\frac{1}{2}\times 36^2\right)\times 12 + \left(-\frac{1}{2}\times 18^2\right)(9+6)}{\left(\frac{1}{2}\times 36^2\right) - \left(\frac{1}{2}\times 18^2\right)} = \frac{2^2\times 12 - 15}{2^2 - 1^2} = \frac{33}{3} = 11$$
