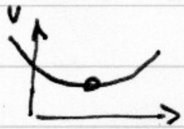
M2 JAN 2010

1. (P) $\longrightarrow$ $v = 3t^2 - 4t + 3$

$$s = \int v\,dt = t^3 - 2t^2 + 3t + c$$

$$s = 0 \text{ at } t = 0 \therefore c = 0$$



Minimum velocity when $dv/dt = 0$

$\frac{dv}{dt} = 6t - 4$, min when $6t - 4 = 0$,

$t = 4/6 = 2/3$ sec.

Then $s = (2/3)^3 - 2(2/3)^2 + 3(2/3) = \frac{38}{27} = 1.407$ m

2. before — positive velocity $\longrightarrow$ — after

	before			after	
velocity	$2u$	$-u$		v_p	v_q
	(P)	(Q)		(P)	(Q)
mass	$2m$	m			

(a) Momentum: $(2m)(2u) - mu = 2mv_p + mv_q$

$(\div m)$ $\therefore 3u = 2v_p + v_q$

Restitution: $e = \frac{v_q - v_p}{2u - (-u)} = \frac{v_q - v_p}{3u}$

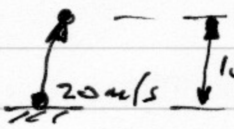
$\therefore 3ue = v_q - v_p$,

Eliminating v_q:

$(2v_p + v_q) - (v_q - v_p) = 3u - 3ue$

$3v_p = 3u(1-e)$, $\quad v_p = u(1-e)$

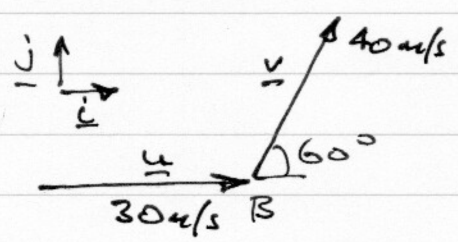
(b) $v_q = 3ue + v_p = u(3e + 1 - e) = u(1 + 2e)$

3. 

Increase in PE = drop in KE − frictional work.

$mg \times 10 = \frac{1}{2}m(20^2 - 0^2) - 10R$

$m = 0.5$ $\therefore 49 = 100 - 10R$, $\quad R = 5.1$ N

4(a) 

$\underline{u} = \begin{bmatrix} 30 \\ 0 \end{bmatrix}$ m/s.

$\underline{v} = \begin{bmatrix} 40\cos 60 \\ 40\sin 60 \end{bmatrix} = \begin{bmatrix} 20 \\ 20\sqrt{3} \end{bmatrix}$ m/s

$m = 0.25$ kg.

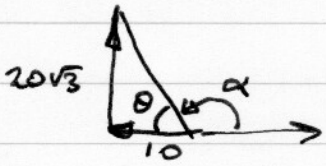
Impulse $I = mv - mu = 0.25\left(\begin{bmatrix}20\\20\sqrt{3}\end{bmatrix} - \begin{bmatrix}30\\0\end{bmatrix}\right)$

$= 0.25\begin{pmatrix}-10\\20\sqrt{3}\end{pmatrix}$ Ns.

Magnitude $= 0.25\sqrt{10^2 + (20\sqrt{3})^2}$

$= 9.014$ Ns $= \underline{9.01 \text{ Ns}}$ to 3 sig. figs.

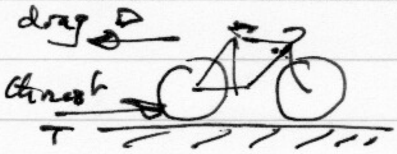
(b)



$\tan\theta = \frac{20\sqrt{3}}{10} = 2\sqrt{3}$, $\theta = 73.9°$

$\therefore \alpha = 180 - 73.9 = 106.1°$ from direction $\vec{AB}$.

5(a) Power = force × speed, $490 = T \times 3.5$, $T = 140$ N

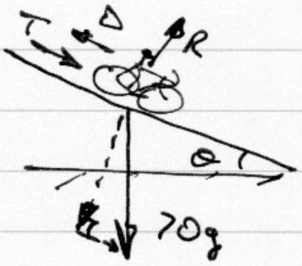


Constant speed $\therefore a = 0$

$T - D = 0$,

$D = T = 140$ N.

(b)



Drag $D = 40u$

Constant speed u

$\therefore T + 70g\sin\theta - D = ma = 0$

$Tu = 24$ watts, $T = \frac{24}{u}$

$\therefore \frac{24}{u} + \frac{70 \times 9.8}{14} - 40u = 0$

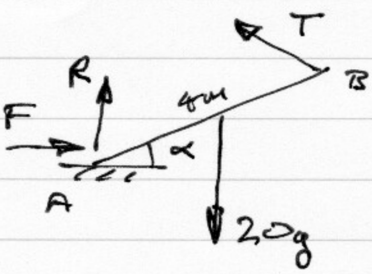
($\times u$) $24 + 49u - 40u^2 = 0$

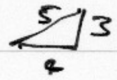
$40u^2 - 49u - 24 = 0$ $ac = -960 = -64 \times 15$

$(8u + \frac{15}{5})(5u - \frac{64}{8}) = (8u+3)(5u-8) = 0$, $u = -3/8$ (silly), $8/5$

$\therefore u = 1.6$ m/s

6.





$\sin\alpha = 3/5$, $\cos\alpha = 4/5$

We don't know T, so take moments about B ↺ :

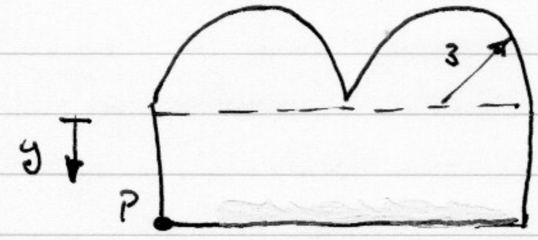
$4\cos\alpha R - 4\sin\alpha F - 20g(2\cos\alpha) = 0$

Limiting equilibrium $\therefore F = F_{max} = \mu R = 0.5R$

$$\frac{16}{5}R - \frac{12}{5}(0.5R) = 40 \times 9.8\left(\tfrac{4}{5}\right)$$

$\therefore 2R = 313.6, \quad R = 156.8\ N$

7(a)

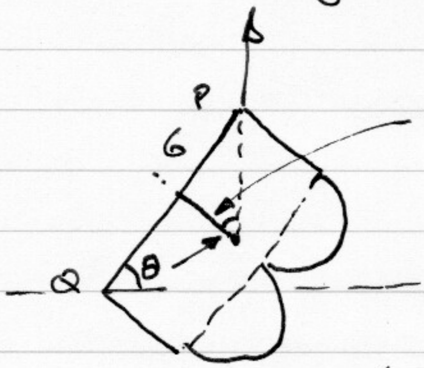


Define y downwards from SR (since we are using "x" already).

$$\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{(24x)x - 2\left(\tfrac{1}{2}\pi 3^2\right) \times \frac{4\times 3}{3\pi}}{24x + 2\left(\tfrac{1}{2}\pi 3^2\right)}$$

$$= \frac{24x^2 - 36}{24x + 9\pi} = \frac{8x^2 - 12}{8x + 3\pi} = \frac{4(2x^2 - 3)}{8x + 3\pi}$$

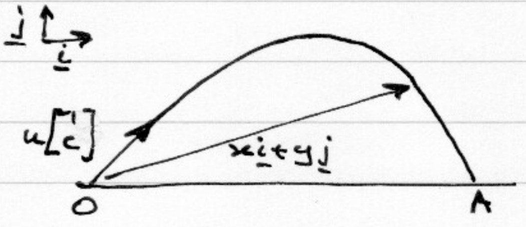
(b) $x = 2 \quad \therefore \bar{y} = \dfrac{4(8-3)}{16 + 3\pi} = \dfrac{20}{16 + 3\pi}$



$$2x - \bar{y} = 4 - \frac{20}{16 + 3\pi} = \frac{64 + 12\pi - 20}{16 + 3\pi}$$

$$= \frac{44 + 12\pi}{16 + 3\pi}$$

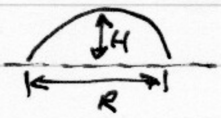
$$\tan\theta = \frac{6}{\left(\frac{44 + 12\pi}{16 + 3\pi}\right)} = \frac{6(16 + 3\pi)}{44 + 12\pi} = \frac{48 + 9\pi}{22 + 6\pi}$$

8 (a) 

$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$ in general.

$$\begin{bmatrix}x\\y\end{bmatrix} = u\begin{bmatrix}1\\c\end{bmatrix}t + \frac{1}{2}\begin{bmatrix}0\\-9.8\end{bmatrix}t^2 \Rightarrow \begin{array}{l} x = ut \\ y = uct - 4.9t^2 \end{array}$$

$t = \frac{x}{u} \quad \therefore \; y = uc\left(\frac{x}{u}\right) - 4.9\left(\frac{x}{u}\right)^2 = cx - \frac{4.9x^2}{u^2}$

(b) 

$u = 7 \;\therefore\; y = cx - \frac{4.9x^2}{49} = cx - 0.1x^2$

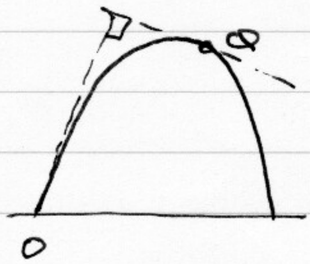
(i) At $y = 0$, $cx - 0.1x^2 = 0$, $x(c - 0.1x) = 0$,

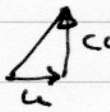
$x = 0$, $x = 10c$ $\quad\therefore\; \underline{R = 10c}$

(ii) At max. height, $dy/dx = 0$

$c - 0.2x = 0$, $\quad x = 5c$

Then $y = c(5c) - 0.1(5c)^2 = 2.5c^2 = H$

(c) 

Gradient at O 

$m_O = c$

$\therefore$ Need gradient at Q $\quad m_Q = \frac{-1}{m_O} = \frac{-1}{c}$

$dy/dx = c - 0.2x$, so at Q $\quad c - 0.2x = \frac{-1}{c}$

$c^2 - 0.2cx + 1 = 0$, $\quad c^2 + 1 = 0.2cx$,

$x = \frac{c^2+1}{0.2c} = \frac{5(c^2+1)}{c}$