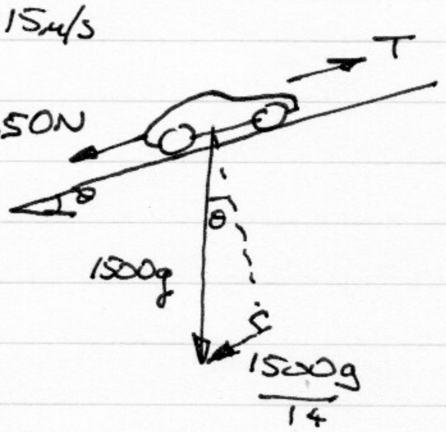
M2 JAN 2009

1. $v = 15\,m/s$



Power = force × speed

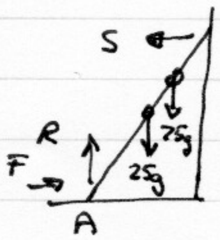
$T = \frac{30000}{15} = 2000\,N$

Resolve ↗:

$T - 650 - \frac{1500g}{14} = ma$

$a = \left(2000 - 650 - \frac{1500 \times 9.8}{14}\right) / 1500 = 0.2\,m/s^2$

2. (a)



Resolving ↑: $R - 25g - 75g = 0$, $R = 100g$.

Limiting equilibrium ∴ $F = F_{max} = \mu R$

$F = \frac{11}{25} \times 100g = 44g = 431.2\,N$

(b) Moments ↺ about A:

$S(4\sin B) - 25g(2\cos B) - 75g(2.8\cos B) = 0$

$S = F = 44g$.

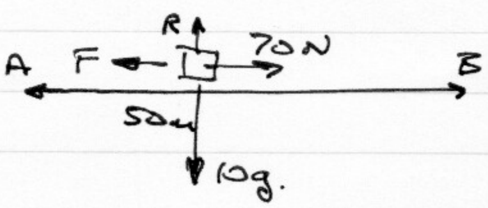
∴ $168g \sin B = 260g \cos B$

$\tan B = \frac{\sin B}{\cos B} = \frac{260g}{168g} = \frac{65}{42}$, $B = 57.13°$

$= 57°$ to nearest °.

(c) His weight is acting at a single point (feet), not for instance hands & feet.

3 (a)



$R = 10g$.

$F = F_{max} = \mu R = \frac{4}{7} \times 10g$

$= 56N$

(b) Work done against friction $= 56 \times 50 = 2800\,J$

$KE_{final} = KE_{initial} +$ net work input

$\frac{1}{2}m(v^2 - u^2) = 50(70 - 56) = 700\,J$

$v^2 = u^2 + 140 = 144$, $v = 12\,m/s$

4. $v = \begin{cases} 10t - 2t^2 & 0 \le t \le 6 \\ -432t^{-2} & t > 6 \end{cases}$

(a) $s = \int v\,dt = 5t^2 - \frac{2}{3}t^3 + c$

At $t=0$, $s=0$ $\therefore c = 0$

At $t=6$, $s = 5(6^2) - \frac{2}{3}(6^3) = 36\,m$

(b) For $t > 6$,

$$s = \int -432t^{-2}\,dt = 432t^{-1} + c$$

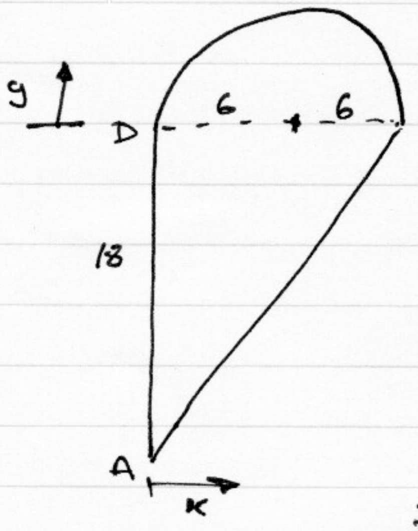
At $t=6$, $s=36$

$$\therefore \quad c = -\frac{432}{6} + 36 = -36$$

At $t=10$,

$$s = \frac{432}{10} - 36 = 7.2\,m$$

5. (a)



$$\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$

Semicircle: $\bar{x} = 6$, $m \propto \frac{\pi 6^2}{2} = 18\pi$

Triangle: $\bar{x} = \frac{0+0+12}{3} = 4$,

$m \propto \frac{1}{2}(12 \times 18) = 108$

$$\bar{x} = \frac{(18\pi)6 + 108 \times 4}{18\pi + 108} = 4.6873$$

$= 4.69$ to 3 s. figs

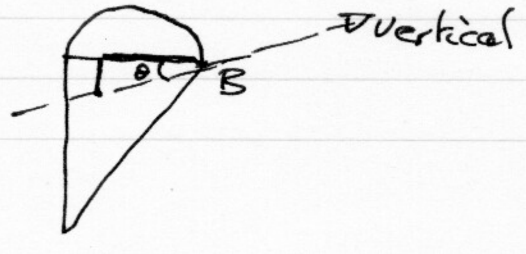
b) Semicircle: $\bar{y} = \frac{4r}{3\pi} = \frac{8}{\pi}$

Triangle: $\bar{y} = -\left(\frac{0+0+18}{3}\right) = -6$

$$\bar{y} = \frac{(18\pi)\frac{8}{\pi} - 108 \times 6}{18\pi + 108} = -3.063,$$

$\therefore$ 3.06 cm below BD

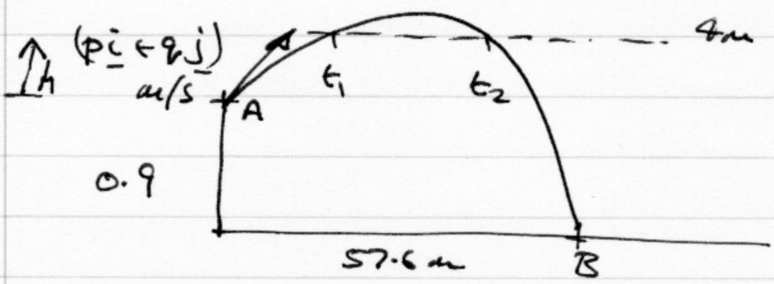
(c)



$$\tan\theta = \frac{3.06}{12 - 4.69} = 0.4189,$$

$\theta = 22.7^\circ$, $= 23^\circ$ to nearest degree.

6.



a) Constant horizontal velocity $s_x = u_x t$,

$$57.6 = 3p, \quad p = 19.2$$

b) Height h above A:

$$h = qt - \tfrac{1}{2}gt^2$$

$$-0.9 = 3q - 4.9(3^2), \quad 3q = 9 \times 4.9 - 0.9$$

$$\therefore q = 14.4$$

(c) Speed $= \sqrt{p^2+q^2} = \sqrt{19.2^2 + 14.4^2} = 24\text{ m/s}$

(d) $\tan\alpha = \dfrac{q}{p} = \dfrac{14.4}{19.2} = \dfrac{3}{4}$

(e)

$h = 14.4t - 4.9t^2 = 4 - 0.9 = 3.1$ when ball 4m above ground.

$$4.9t^2 - 14.4t + 3.1 = 0$$

$$t = \frac{14.4 \pm \sqrt{14.4^2 - 4 \times 4.9 \times 3.1}}{2 \times 4.9} = \frac{14.4 \pm \sqrt{146.6}}{9.8}$$

$$t_2 - t_1 = 2 \times \frac{\sqrt{146.6}}{9.8} = 2.47 \text{ seconds (to 3 s.figs)}$$

(f) Drag due to wind resistance.

7.(a) $\longrightarrow$ pos. velocity.

	P	e	Q
	$3m$		$2m$
before	$2u$		$-u$

	P	Q
after	v_p	v_q

Momentum

$$3m(2u) + 2m(-u) = 3mv_p + 2mv_q$$

$$\therefore \; 4u = 3v_p + 2v_q$$

Restitution: $e = \dfrac{v_q - v_p}{2u-(-u)}$

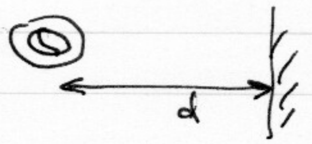
$$\therefore \; v_q - v_p = 3eu, \qquad \left.\begin{aligned} 3v_q - 3v_p &= 9eu \\ 2v_q + 3v_p &= 4u \end{aligned}\right\} +, \quad 5v_q = u(9e+4)$$

$$\therefore \; v_q = \tfrac{1}{5}(9e+4)u$$

(b) $v_p = v_q - 3eu = u\left(\tfrac{9}{5}e - 3e + 4/5\right) = u\left(4/5 - \tfrac{6}{5}e\right)$

$v_p = \tfrac12 u \;\therefore\; \tfrac15(4-6e) = 1/2, \quad 4-6e = 5/2, \quad 6e = 1\tfrac12, \quad e = \tfrac14$

(c)



$$v_q = \tfrac15\left(\tfrac94 + 4\right)u \quad \text{with } e = 1/4$$

$$= \frac{6.25}{5}u = 1\tfrac14 u$$

Q takes time $t = \dfrac{d}{(5/4\,u)} = \dfrac45 \dfrac{d}{u}$ to reach the wall.

In this time, P moving at $1/2\,u$ travels

$(\tfrac12 u)(4/5\; d/u) = 2/5\, d$. It is then $d - 2/5\,d = 3/5\,d$ from the wall.

(d) $e = \dfrac{w_q}{v_q} = 1/5$ (speed w_q after hitting wall).

$\therefore \; w_q = 1/5\left(\tfrac54 u\right) = \tfrac14 u$

Defining t as time after Q hits the wall,

distance of Q from wall $= \tfrac14 ut$

distance of P from wall $= 3/5\,d - 1/2\,ut$

$\rightarrow$ balls hit when $3/5\,d - 1/2\,ut = 1/4\,ut, \quad 3/5\,d = 3/4\,ut,$

$\tfrac15 d = \tfrac14 ut =$ distance of Q from wall

$\therefore$ B is $1/5\,d$ from the wall.