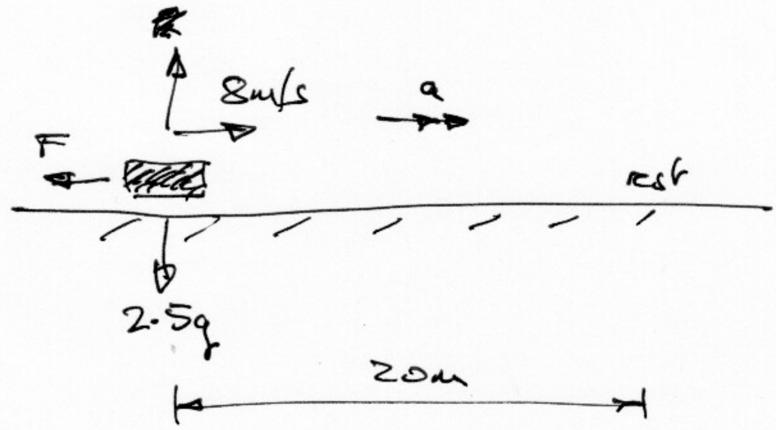
M2 JAN 2008

1.



(a) Initial KE $= \frac{1}{2}mu^2 = \frac{1}{2} \times 2.5 \times 8^2 = 80\text{J}$

Final KE $= 0$

$\therefore$ Energy lost $= 80\text{J}$

(b) Work = force × distance,

$80 = 20F, \quad F = 4\text{N} \quad (= \text{"}R\text{"})$.

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2. $\underline{r} = (3t^2 - 6t + 4)\underline{i} + (3t^3 - 4t)\underline{j}$

(a) $\underline{v} = \frac{d\underline{r}}{dt} = \left[(6t-6)\underline{i} + (9t^2 - 4)\underline{j}\right]$ m/s

(b) Parallel to $\underline{i}$ $\Rightarrow$ zero $\underline{j}$ component

$9t^2 - 4 = 0, \quad t^2 = 4/9, \quad t = \pm 2/3$ seconds

$= 2/3$ sec $\quad (t \geq 0)$.

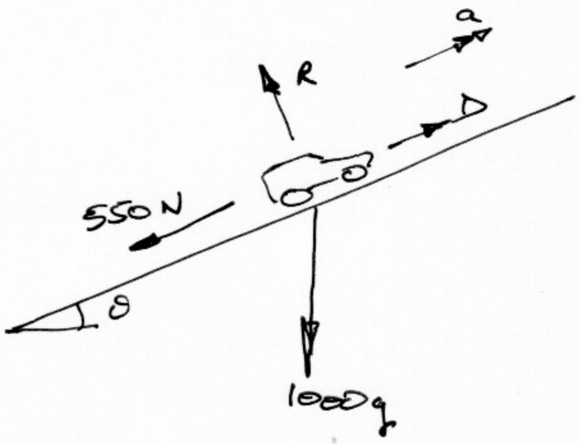
(c) At $t = 1$, $\underline{u} = 0\underline{i} + 5\underline{j}$ m/s before impulse

$\underline{I} = m\underline{v} - m\underline{u}, \quad \underline{v} = \underline{u} + \underline{I}/m$

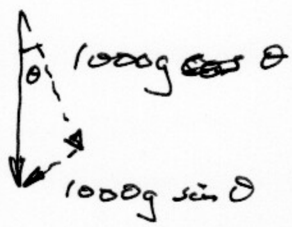
$= 5\underline{j} + \frac{2\underline{i} - 6\underline{j}}{0.5} = (4\underline{i} - 7\underline{j})$ m/s

after impulse

3.



Components of 1000g:



(a) Resolving up slope →, constant speed ($a = 0$):

$$D - 550 - 1000g \sin\theta = 0$$

Driving force $D = \dfrac{20000}{16} = 1250\text{N}$

$$\sin\theta = \frac{1250 - 550}{1000g} = \frac{0.7}{9.8} = \frac{1}{14}$$

(b) Now $D = 0$

$$-550 - 1000g\sin\theta = ma = 1000a$$

$$g\sin\theta = 0.7$$

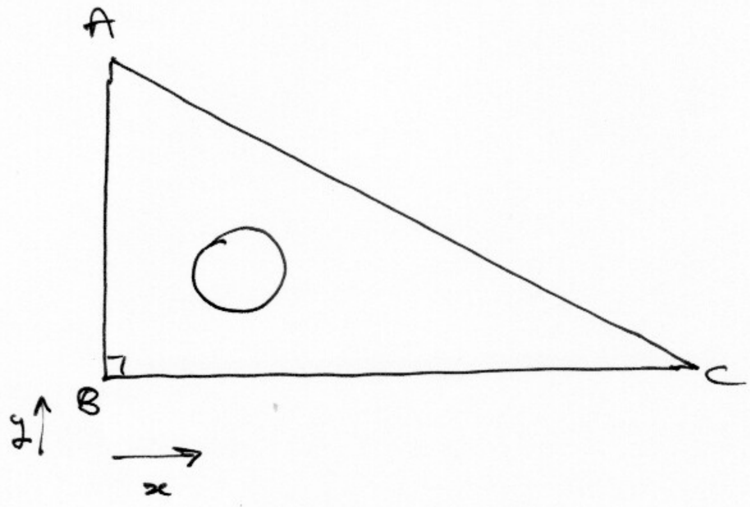
$$\therefore\ a = -0.55 - 0.7 = -1.25\text{m/s}^2$$

$$v^2 = u^2 + 2as$$

$$0^2 = 16^2 + 2(-1.25)s, \qquad s = \frac{256}{2.5} = 102.4\text{m}$$

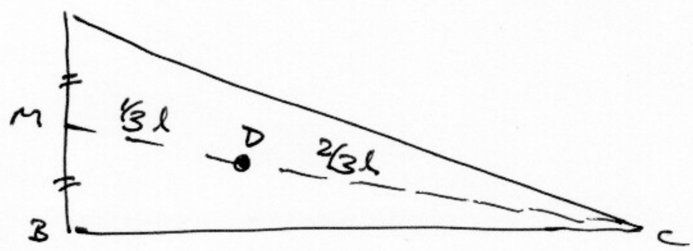
($=y$)

4.



(a)

Centre of mass of triangle (complete) at (7, 4)
(= 1/3 of way up) because (eg):



M is mid-point of AB ie 6 cm up
D is 2/3 of way from C to M so $\frac{2}{3} \times 6 = 4$ cm up.

Area of triangle $= \frac{1}{2} \times 12 \times 21 = 126\ \text{cm}^2$

Area of cut-out $= \pi r^2 = 9\pi\ \text{cm}^2$

The hole is a "negative mass"

	A_i	x_i	$A_i x_i$	y_i	$A_i y_i$
Triangle	126	7	882	4	504
Hole	-9π	5	-45π	5	-45π
Totals	$126-9\pi$		$882-45\pi$		$504-45\pi$

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{882-45\pi}{126-9\pi} = 7.579\ \text{cm}$$

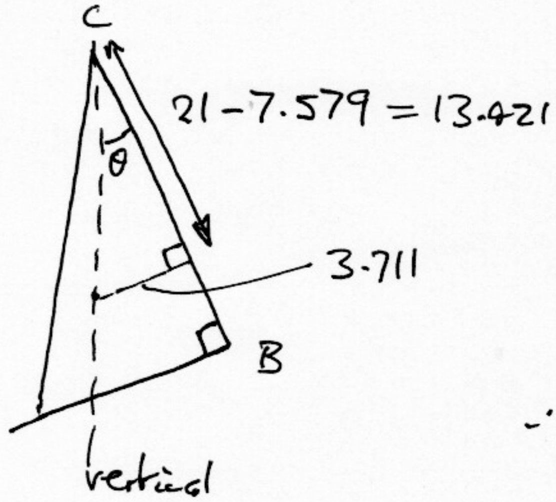
$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{504-45\pi}{126-9\pi} = 3.711\ \text{cm}$$

so (i) 7.579 cm from AB,

(ii) 3.711 cm from BC

[check : the hole moves the C of M to right ✓ and down ✓.]

(b)

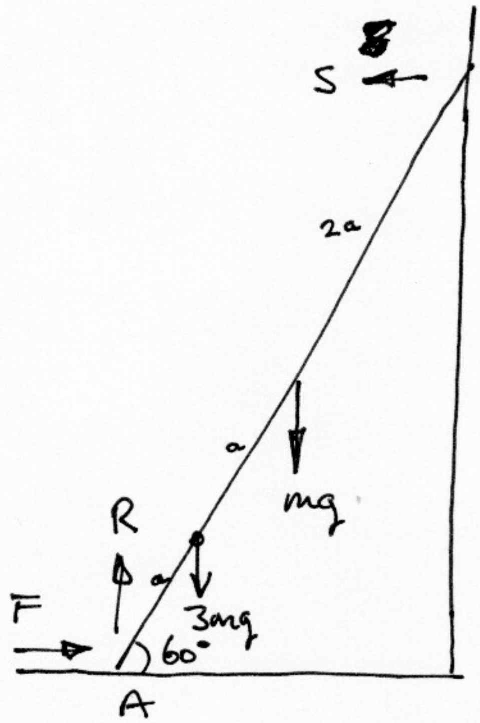


$\tan\theta = \dfrac{3.711}{13.421} = 0.2765$

$\therefore \theta = 15.46^\circ$

$= 15^\circ$ to 0 decimal places.

5.



Moments about A:

$$4aS\sin 60 - 2amg\cos 60 - a(3mg)\cos 60 = 0$$

$$S = \frac{5amg\cos 60}{4a\sin 60} = \frac{5}{4}\frac{mg}{\sqrt{3}}$$

Horizontal equilibrium: $F = S$

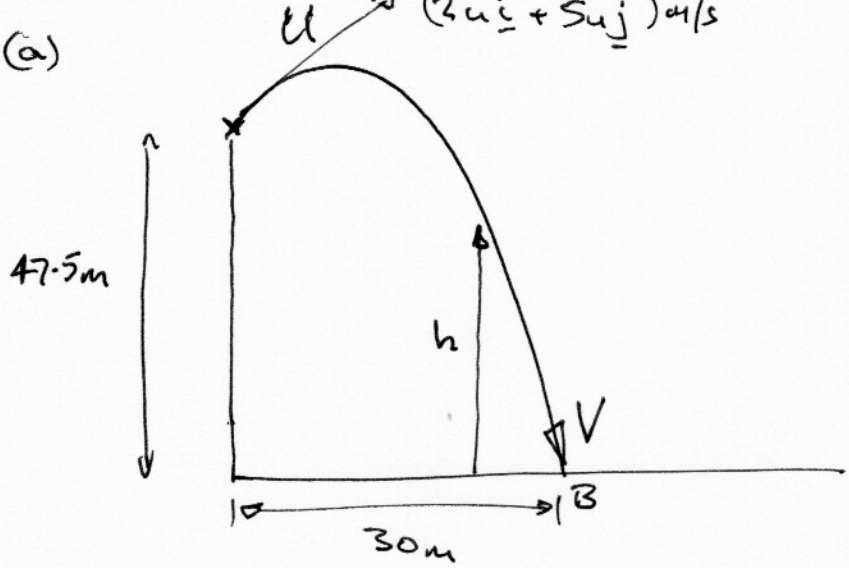
Vertical forces: $R = 4mg$

Since in limiting equilibrium, $\mu = \frac{F}{R}$

$$\therefore \mu = \frac{\frac{5}{4}\frac{mg}{\sqrt{3}}}{4mg} = \frac{5}{16\sqrt{3}} = 0.1804$$

$= 0.180$ to 3 sig. figs.

6. (a)



<u>Horizontal motion (constant speed →)</u>

$2ut = 30, \quad t = \frac{30}{2u}$ at B

$= \frac{15}{u}$.

<u>Vertical motion</u>

$h = 47.5 + u_y t + \frac{1}{2} a_y t^2$

$= 47.5 + 5ut - 4.9t^2 = 0$ at B

$ut = 15$ from before, so

$4.9t^2 = 47.5 + 5 \times 15 = 122.5$, $t^2 = 25$,

<u>$t = 5$ seconds</u>.

(b) $u = \frac{15}{t} = 3$

(c) Initial speed $U^2 = (2^2+5^2)u^2 = (4+25)3^2 = 261$

Then $V^2 = U^2 + 2as = 261 + 2 \times 9.8 \times 47.5 = 1192$

distance in direction of force

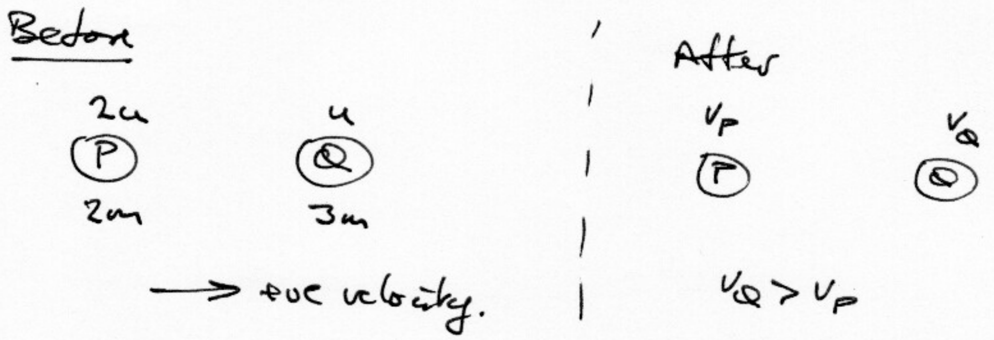
$V =$ speed at $B = \sqrt{1192} = 34.5$ m/s (to 3 s.f.)

[or Vertically $v_y^2 = (5u)^2 + 2 \times 9.8 \times 47.5 = 1156$

$V^2 = 1156 + (2u)^2 = 1156 + 36 = 1192$]

7.

(a)



Momentum conservation:

$$2m(2u) + 3m(u) = 2mV_P + 3mV_Q$$

Restitution:

$$\frac{V_Q - V_P}{2u - u} = e = \tfrac{1}{2}, \quad V_Q = V_P + \tfrac{1}{2}u$$

$$4mu + 3mu = 7mu = 2mV_P + 3m\,(V_P + \tfrac{1}{2}u)$$

$$= 5mV_P + \tfrac{3}{2}mu$$

$$\therefore\ 5\tfrac{1}{2}u = 5V_P, \qquad V_P = \tfrac{11}{10}u$$

$$V_Q = V_P + \tfrac{1}{2}u = \tfrac{16}{10}u = \tfrac{8}{5}u$$

(b) Kinetic energy loss = initial KE − final KE

$$= \tfrac{1}{2}(2m)(2u)^2 + \tfrac{1}{2}(3m)u^2 - \tfrac{1}{2}(2m)\left(\tfrac{11}{10}u\right)^2 - \tfrac{1}{2}(3m)\left(\tfrac{8}{5}u\right)^2$$

$$= \left(4 + 1\tfrac{1}{2} - 1.21 - \tfrac{3}{2}\times 2.56\right)mu^2$$

$$= \tfrac{9}{20}mu^2$$

7(c) $\frac{11}{10}u$ before $8/5u$ 0

P Q $3m$ R m

after V_{Q2} V_{R2} $(V_{R2} > V_{Q2})$

The second impact reduces Q's velocity. It will hit P if its velocity is less than $\frac{11}{10}u$ (P catches it up).

Momentum:

$$3m(8/5u) = 3mV_{Q2} + mV_{R2}$$

Restitution:

$$\frac{V_{R2} - V_{Q2}}{8/5u} = e$$

If $e = 0$, $V_{R2} = V_{Q2} =$ (from momentum) $\frac{3}{4}\frac{8}{5}u$

$$= \frac{6}{5}u = \frac{12}{10}u \text{ , so } > V_P .$$

As e increases, V_{Q2} reduces.

If $V_{Q2} = \frac{11}{10}u$:

$$3m(8/5u) = 3m\left(\frac{11}{10}u\right) + mV_{R2}$$

$$3\left(\frac{16}{10} - \frac{11}{10}\right)u = 3\left(\frac{5}{10}\right)u = \frac{3}{2}u = V_{R2}$$

$$\therefore e = \frac{\frac{3}{2}u - \frac{11}{10}u}{8/5u} = \frac{0.4}{1.6} = 0.25$$

so for $0.25 < e \leq 1$ there will be a second P-Q collision.