Jan 07 }
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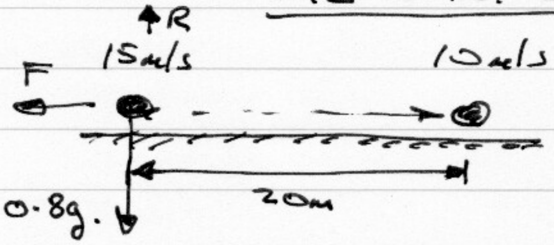
M2 JAN. 2007

1. (a)



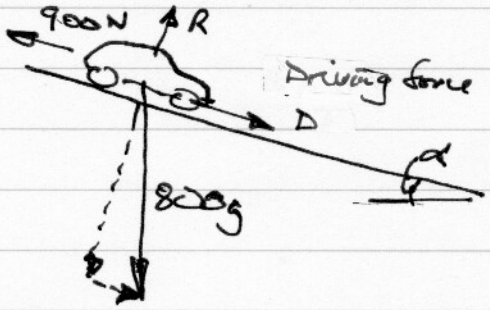
Resolve ↑ $R - 0.8g = 0$,
$R = 0.8g$.

Work done against friction = loss of KE
$= \frac{1}{2}m(15^2) - \frac{1}{2}m(10^2)$
$= \frac{0.8}{2}(225-100) = 50\,J$

nb Not "work done by friction" – silly question, engines do work, friction absorbs it.

(b) Fd = work, $F = \frac{50}{20} = 2.5\,N$
sliding ∴ $F = F_{max}$, $\mu = F/R = \frac{2.5}{0.8g} = 0.319$

2(a)



constant speed ∴ resolve ↘,
$D + 800g\sin\alpha - 900 = ma = 0$

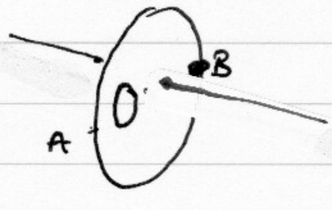
$D = 900 - \frac{800 \times 9.8}{24} = \frac{1720}{3}$
$= 573.3\,N$

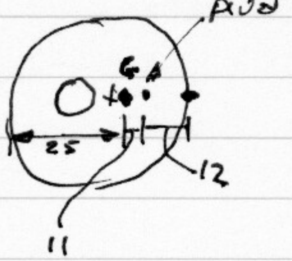
Power = force × speed $= 573.3 \times 15 = 8600\,W = 8.6\,kW$.

(b) Now $D = 0$, $800g\sin\alpha - 900 = 800a$
$a = g\sin\alpha - \frac{900}{800} = -\frac{43}{60} = -0.717\,m/s^2$
$v = u + at$,
$0 = 15 + (-0.717)T$, $T = 20.93$ sec
≈ 21 sec.

3 (a) $\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{(\pi 24^2)24 - (\pi 8^2)16}{\pi 24^2 - \pi 8^2} = \frac{3^2 \times 24 - 16}{3^2 - 1} = 25$ cm from A.

(b)





Let M = mass of T
$11M = 12(\frac{1}{4}m)$
$M = \frac{3m}{11}$

4(a) <u>Before</u> | <u>After</u> $\longrightarrow$ pos. velocity

u	0	$-v$	$3v$
P	Q	P	Q
m	km	m	km

<u>Restitution</u> $e = \frac{1}{2} = \frac{\text{separation speed}}{\text{approach speed}} = \frac{3v-(-v)}{u} = \frac{4v}{u}$

$\therefore \frac{1}{2}u = 4v, \quad u = 8v$

(b) <u>Momentum</u> $mu + 0 = m(-v) + km(3v)$

$u = -v + 3kv = (3k-1)v$

$\therefore 8v = (3k-1)v, \quad 3k-1 = 8, \quad 3k = 9, \quad k = 3$

(c) <u>before</u> | after

$3v$	0	$-w$	w
Q	R	Q	R
km $= 3m$	$11m$		

<u>Restitution</u>: $\frac{w-(-w)}{3v} = e$

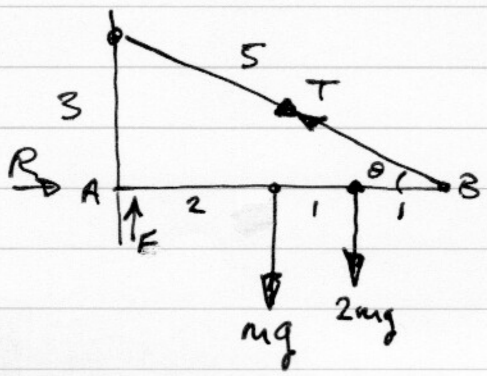
<u>Momentum</u>: $3m(3v) + 0 = 3m(-w) + 11m(w) = 8mw$

$\therefore w = \frac{9v}{8}$

$e = \frac{2w}{3v} = \frac{(\frac{9}{4}v)}{3v} = 3/4$

(d) P is now moving at $-v$ (ie. v to left $\leftarrow$).
Q is moving at $-\frac{9}{8}v$ (ie. $\frac{9}{8}v$ to left $\leftarrow$, faster, catches up with P) $\therefore$ collides again.

5(a)



$\sin\theta = 3/5 \quad \cos\theta = 4/5$

<u>Moments about A</u> ↺

$T(4a\sin\theta) - mg(2a) - 2mg(3a) = 0$

$T = \frac{8mga}{4a(3/5)} = 2 \times 5/3\, mg$

$= \frac{10}{3}mg$

(b) <u>Resolve</u> $\rightarrow$ $R - T\cos\theta = 0, \quad R = 4/5\, T = \frac{8}{3}mg$

(c) <u>Resolve</u> ↑ $F - mg - 2mg + T\sin\theta = 0$

$$F = 3mg - \left(\tfrac{10}{3}mg\right)\tfrac{3}{5} = 3mg - 2mg = mg$$

Limiting equilibrium

$$\therefore F = F_{max}, \quad \mu = F/R = \frac{mg}{\tfrac{8}{3}mg} = 3/8$$

6(a) $\underline{F} = \left[(1.5t^2 - 3)\underline{i} + 2t\underline{j}\right]$ N

$$\underline{F} = m\underline{a}, \quad \underline{a} = \frac{F}{m} = \frac{(1.5t^2-3)i + 2tj}{0.5} = \left((3t^2-6)\underline{i} + 4t\underline{j}\right) \text{ m/s}^2$$

(b) Acceleration is <u>not constant</u>, cannot use $v = u + at$ etc.

integrate

$$v = u + \int_2^3 \underline{a}\,dt = \begin{pmatrix}-4\\5\end{pmatrix} + \begin{bmatrix}t^3 - 6t\\2t^2\end{bmatrix}_2^3$$

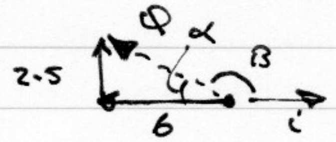
$$= \begin{pmatrix}-4\\5\end{pmatrix} + \begin{bmatrix}(27-18)-(8-12)\\18-8\end{bmatrix} = \begin{bmatrix}-4\\5\end{bmatrix} + \begin{bmatrix}13\\10\end{bmatrix} = \begin{bmatrix}9\\15\end{bmatrix}$$

$\therefore v = [9i + 15j]$ m/s at $t = 3$.

(c) $\underline{Q} = mv - mu = 0.5\left(\begin{pmatrix}-3\\20\end{pmatrix} - \begin{pmatrix}9\\15\end{pmatrix}\right) = 0.5\begin{pmatrix}-12\\5\end{pmatrix} = \begin{pmatrix}-6\\2.5\end{pmatrix}$

$|\underline{Q}| = \sqrt{6^2 + 2.5^2} = 6.5$ Ns

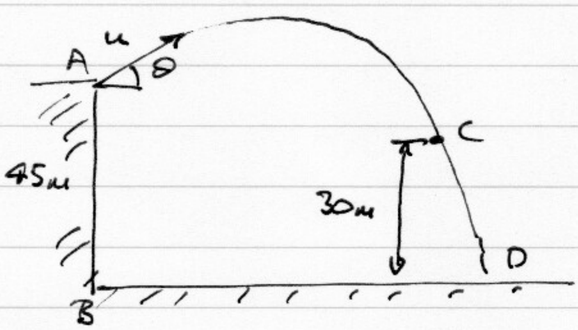
(d)



$\tan\alpha = 2.5/6, \quad \alpha = 22.6°,$

$\beta = 180 - 22.6 = 157.4°$

7.

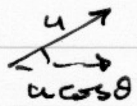


(a) $v^2 = u^2 + 2as$, $\quad 24.5^2 = u^2 + 2 \times 9.8 \times (45-30) = u^2 + 294$

in same direction!

$\therefore u^2 = 306.25$, $\quad u = 17.5\,m/s$

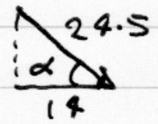
(b)



Horizontal component (constant)

$u\cos\theta = 17.5 \times 4/5 = 14\,m/s$

At C :

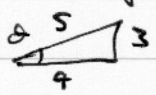


$\cos\alpha = \frac{14}{24.5} = \frac{4}{7}$,

$\alpha = \cos^{-1}(4/7) = 55.15^\circ$

(c) Height above starting point $\quad s = (u\sin\theta)t + \frac{1}{2}at^2$

Hits ground when $\quad (17.5\sin\theta)t + \frac{1}{2}(-9.8)t^2 = -45$



$\sin\theta = 3/5$

$\therefore 10.5t - 4.9t^2 = -45, \quad 4.9t^2 - 10.5t - 45 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10.5 \pm \sqrt{10.5^2 + 4 \times 4.9 \times 45}}{9.8} = \frac{30}{7}, \quad \frac{-15}{7}$

(or $49t^2 - 105t - 450 = 0$, $\quad ac = 49 \times -450 = 105 \times -210$

$(7t + \frac{105}{7})(7t - \frac{210}{7}) = 0 \quad , \quad (7t+15)(7t-30) = 0$).

Horizontal distance $= (u\cos\theta)t = (17.5 \times 4/5)(\frac{30}{7}) = 60\,m$