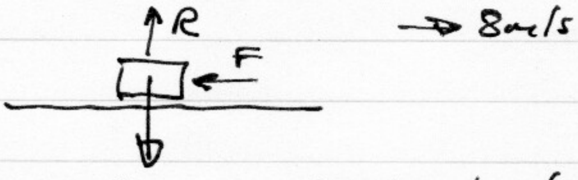
M2 JAN 2006

1.(a)



$\Delta KE = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 3(0 - 8^2) = -96\,J$

$\therefore$ loss in KE is 96 J.

(b) Work = force × distance, $F = W/d = \frac{96}{12} = 8\,N$

$R - 3g = 0$, $R = 3 \times 9.8 = 29.4\,N$.

Sliding $\therefore F = F_{max}$, $\mu = F/R = 8/29.4 = 0.272$

2. $\underline{r} = [(t^2 + 4t)i + (3t - t^3)j]$ m

(a) $\underline{v} = \frac{dr}{dt} = [(2t + 4)\underline{i} + (3 - 3t^2)\underline{j}]$ m/s

At $t = 3$, $\underline{v} = (6 + 4)i + (3 - 27)j = (10i - 24j)$ m/s

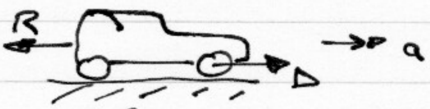
$\therefore$ speed $= \sqrt{10^2 + 24^2} = 26$ m/s

(b) $I = mv - mu = m(v - u)$

$\therefore v = \frac{I}{m} + u = \frac{8i - 12j}{0.4} + (10i - 24j)$

$= (20i - 30j) + (10i - 24j) = [30\underline{i} - 54\underline{j}]$ m/s

3(a)



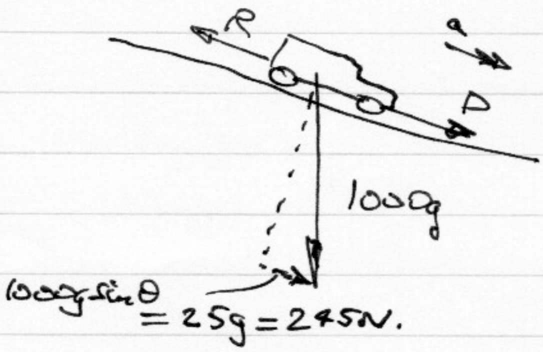
Equation of motion →

$D - R = ma$

Power = force × speed, $12000\,W = D \times 15\,m/s$, $D = 800\,N$

$R = D - ma = 800 - 1000 \times 0.2 = 600\,N$

(b)



"$\Sigma F = ma$"

$D + 25g - R = 0$

$D = 600 - 245 = 355\,N$

Power = force × speed

$7000 = 355 \times u$, $u = 19.7$ m/s

4.(a) <u>Before</u> | <u>After</u> → positive vel.

(A)	(B)	(A)	(B)
$2m$	m	$2m$	m
$3u$	$-2u$	v_A	$\frac{8}{3}u$

<u>Momentum</u>: $2m(3u) + m(-2u) = 2mv_A + m(\frac{8}{3}u)$

$(\div m)$ $\quad 6u - 2u = 2v_A + \frac{8}{3}u, \quad v_A = \frac{1}{2}(4u - \frac{8}{3}u) = \frac{2}{3}u$

$$\text{Coeff. of restitution} = \frac{\text{separation speed}}{\text{approach speed}} = \frac{\frac{8}{3}u - \frac{2}{3}u}{3u-(-2u)} = \frac{2u}{5u} = \frac{2}{5}$$

(b) $\Delta KE = KE_{final} - KE_{initial} = \frac{1}{2}\left[\left(2m(\frac{2}{3})^2 + m(\frac{8}{3})^2\right) - \left(2m(3^2) + m(-2)^2\right)\right]u^2$

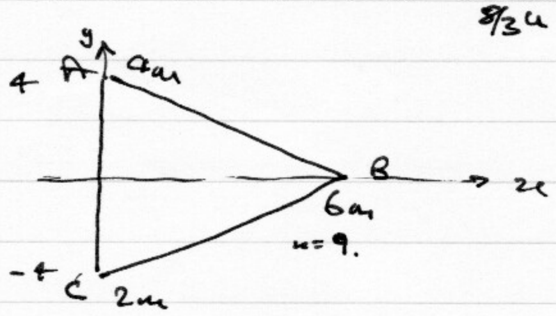
$= \frac{1}{2}mu^2\left(\frac{8}{9} + \frac{64}{9} - 18 - 4\right) = -7mu^2 \quad \therefore$ Loss of $KE = 7mu^2$.

(c) Let B have velocity U before impact, V afterwards.

$mV = mU - \frac{14}{3}mu, \quad V = U - \frac{14}{3}u = \left(\frac{8}{3} - \frac{14}{3}\right)u = -2u$

$$\text{Coeff. of restitution} = \frac{2u}{\frac{8}{3}u} = 2 \times \frac{3}{8} = \frac{3}{4}.$$

5.



a) $\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{(6\times 9 + 4\times 0 + 2\times 0)m}{(6+4+2)m} = \frac{54}{12} = 4\frac{1}{2}$

$\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{(6\times 0 + 4\times 4 + 2(-4))m}{(6+4+2)m} = \frac{8}{12} = \frac{2}{3}$

$\therefore (\bar{x}, \bar{y}) = (4\frac{1}{2}, \frac{2}{3})$

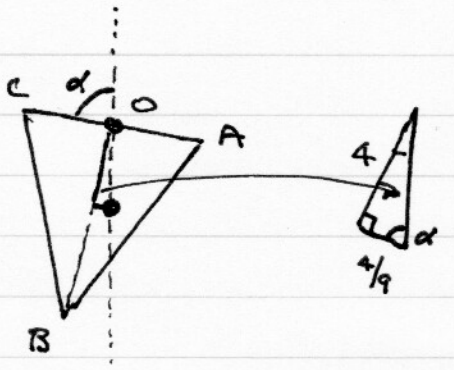
b) <u>Lamina by itself</u>: $\bar{x} = \frac{0+0+9}{3} = 3, \quad \bar{y} = 0$ by symmetry.

3 masses + lamina: $\bar{x} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{6m\times 9 + 0 + 0 + km(3)}{(6+4+2+k)m} = \frac{54 + 3k}{12+k} = 4$

$\therefore 54 + 3k = 48 + 4k, \quad k = 6$

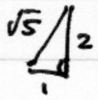
c) $\bar{y} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{(6\times 0 + 4\times 4 + 2(-4) + 6(0))m}{(6+4+2+6)m} = \frac{8}{18} = \frac{4}{9} = \lambda$

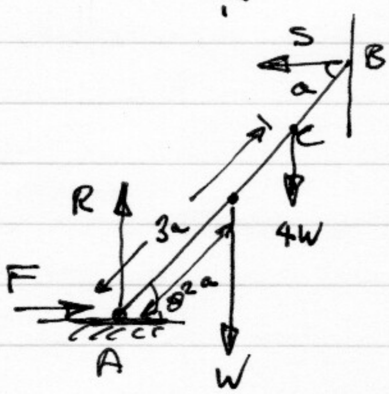
(d)



$\tan\alpha = \frac{4}{(4/9)} = 9$

$\therefore \alpha = 83.66^\circ$

6 (a) $\tan\theta = 2$  , $\sin\theta = \frac{2}{\sqrt5}$, $\cos\theta = \frac{1}{\sqrt5}$.



Moments about A ↺ :

$S(4a\sin\theta) - W(2a\cos\theta) - 4W(3a\cos\theta) = 0$

$4\times\frac{2}{\sqrt5}S - \frac{2}{\sqrt5}W - \frac{12}{\sqrt5}W = 0$

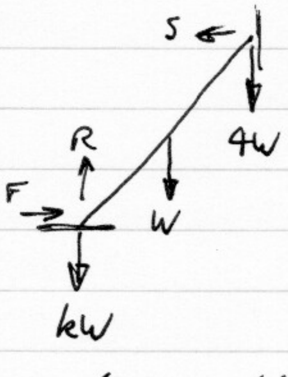
$8S = 14W$, $S = \frac{14}{8}W = \frac{7}{4}W$

Resolve → $F - S = 0$, $F = 7/4\,W$.

Resolve ↑ $R - W - 4W = 0$, $R = 5W$

In limiting equilibrium $\therefore F = \mu R$, $\mu = \frac{F}{R} = \frac{\frac{7}{4}W}{5W} = \frac{7}{20} = 0.35$

(b)



Moments about A ↺

$S(4a\sin\theta) - W(2a\cos\theta) - 4W(4a\cos\theta) = 0$

$\frac{4\times2}{\sqrt5}S - \frac{2}{\sqrt5}W - \frac{16}{\sqrt5}W = 0$, $S = \frac{18}{8}W = \frac{9}{4}W$

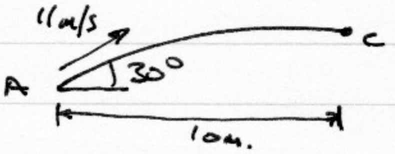
$F = S$.

$R = 5W + kW$

In equilibrium $\therefore F \le \mu R$, $\frac{9}{4}W \le \frac{7}{20}(5+k)W$

$\left(\times\frac{20}{W}\right)$: $45 \le 35 + 7k$, $7k \ge 10$, $k \ge \frac{10}{7}$

7. (a)



<u>Horizontal motion</u> $u = 11\cos 30$ m/s, $a = 0$,

$$t = \frac{10}{11\cos 30} = 1.0497 \text{ sec} = 1.05\text{ s (2dp)}.$$

(b) <u>Vertical motion</u>: $u = 11\sin 30$ m/s, $a = -9.8$ m/s^2

$$s = ut + \tfrac{1}{2}at^2 = 11\sin 30 \times 1.05 - 4.9 \times 1.05^2 = 0.374 \text{ m up.}$$

$\therefore$ $TC = 2 - (1 + 0.374) = 0.626\text{ m} \approx 0.63$ m

(c) $t = \dfrac{10}{V\cos 30}$ (as (a)).

$s = (V\sin 30)t - 4.9t^2 = 1$m so hits pole 1m above A (as (b)).

$$\therefore\ 10\left(\frac{\cancel{V}\sin 30}{\cancel{V}\cos 30}\right) - 4.9\left(\frac{10}{V\cos 30}\right)^2 = 1$$

$$\frac{490}{V^2\cos^2 30} = 10\tan 30 - 1$$

$$V^2\cos^2 30 = \frac{490}{10\tan 30 - 1}, \quad V^2 = 136.87,$$

$V = 11.699 = 11.7$ m/s (to 3 s.fig).

[check: > 11 m/s ✓].

(d) The ball is of finite size, not a particle. Will hit the top if the centre passes $<$ 1 radius above T.