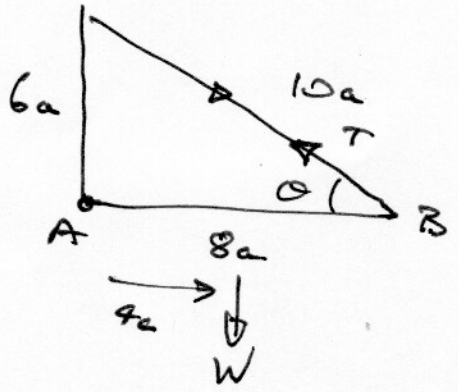
M2 Jan 2005

1.



$\sin\theta = \frac{6}{10} = 3/5.$

$\cos\theta = \frac{8}{10} = 4/5$

a) Moments about A: $8aT\sin\theta - 4aW = 0$

$$T = \frac{4aW}{8a\sin\theta} = \frac{W}{2\times 3/5} = \frac{5W}{6}.$$

b) Horizontal force $= T\cos\theta = 4/5 \cdot \frac{5}{6}W$

$$= \frac{2}{3}W.$$

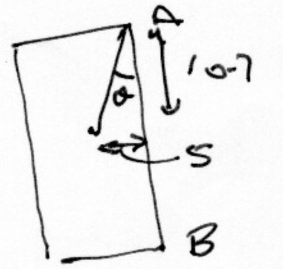
2. $m\bar{x} = m_1\bar{x}_1 - m_2\bar{x}_2$ where m_2 is mass removed

~~200 area~~

$$(200 - 3^2\pi)\bar{x} = 200\times 10 - 3^2\pi\times 6$$

$$\bar{x} = 10.6586 \text{ cm}$$

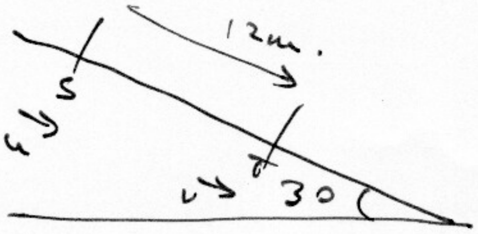
$= ~~10.659~~$ 10.7 cm. to 3 s.f.



$\tan\theta = \frac{5}{10.7} = 0.469,$

$\underline{\theta = 25.1°}$

3 a)



$u = 10\,m/s$

$v = 9\,m/s$

$s = 12\,m$.

a) Loss of PE $= mg\Delta h$

$= 0.6 \times 9.8 \times 12 \sin 30 = 35.28\,J$

Loss of KE $= \frac{1}{2} \times 0.6\,(10^2 - 9^2) = 5.7\,J$

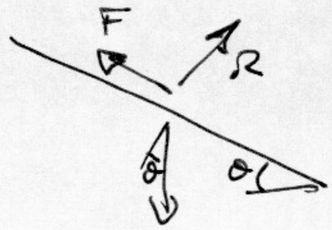
Total $40.98\,J$

$\approx 41.0\,J$.

b) $2as = v^2 - u^2 = \frac{81 - 100}{\text{~~100-81~~}} = 19$

$\therefore a = \frac{-19}{2s} = \frac{-19}{24} = -0.79166\,m/s^2$

Net force $F = ma = -0.79166M\ N = -0.475\,N$.



$Mg \sin\theta - F = ma$

$= -0.475$

~~$\therefore F = 0.6 \times 9.8 \sin 30 - 0.475 = 2.465\,N$~~

$F = mg \sin\theta + 0.475 = 3.415\,N$.

$R = mg \cos\theta = 5.092\,N$

$\therefore \mu = F/R$ (since slipping)

$= 0.6706$

4.

a) $\underline{F} = m\underline{a} = m\frac{d\underline{v}}{dt}$

$= 0.4\left(6\underline{i} + (2t+3)\underline{j}\right)$

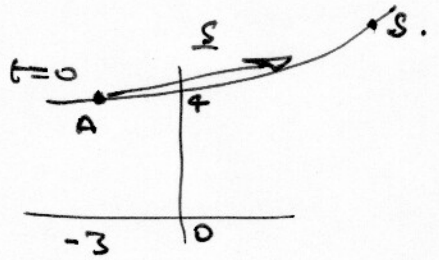
At $t = 4$, $F = 2.4\underline{i} + 4.4\underline{j}$,

magnitude of $F = \sqrt{2.4^2 + 4.4^2} = \sqrt{25.12}$

$= ~~11.26N~~$.

$\underline{5.012 N}$

b)



~~$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}$~~

$\underline{s} = \int_0^4 \underline{v}\, dt = \left[(3t^2 + 4t)\underline{i} + \left(\frac{1}{3}t^3 + \frac{3}{2}t^2\right)\underline{j}\right]_0^4$

$= 64\underline{i} + \left(\frac{64}{3} + 24\right)\underline{j} = 64\underline{i} + 45\frac{1}{3}\underline{j}$

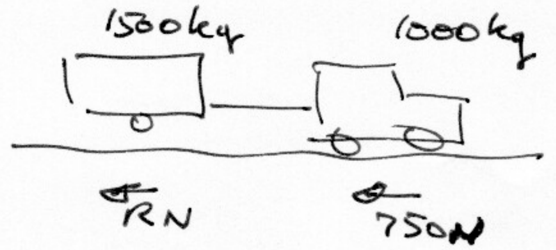
Vector $\underline{OS} = (-3\underline{i} + 4\underline{j}) + (64\underline{i} + 45\frac{1}{3}\underline{j})$

$= 61\underline{i} + 49\frac{1}{3}\underline{j}$

Length $OS = \sqrt{61^2 + 49\frac{1}{3}^2}$

$= 78.45\,m$.

5.



a) Constant speed $\Rightarrow$ $50kW = (R+750)V$

$V = 25 m/s$.

$R + 750 = \frac{50000}{25} = 2000$

$\therefore R = 2000 - 750 = 1250 N$.

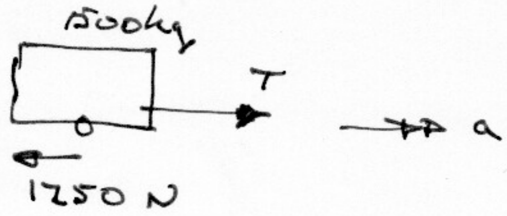
b) Total drag = $1250 + 750 + 1500 = 3500 N$

$-3500 = ma = 2500a$,

$a = \frac{-3500}{2500} = \frac{-7}{5} = -1.4 m/s^2$

(or deceleration of $1.4 m/s^2$)

c)



Eq'n of motion:

$T - 1250 = 1500a$, $a = -1.4$

$T = 1500 \times -1.4 + 1250$

$= -850 N$.

(850 N thrust).

d) Work done = loss of KE

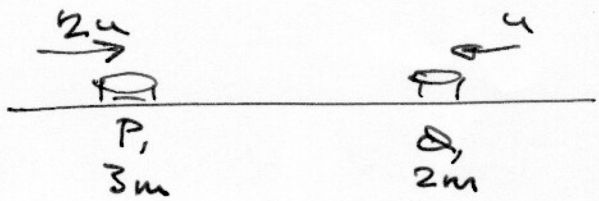
$= \frac{1}{2} \times 2500 \times 25^2 = 781250 J$

$= 781.25 kJ$.

& $\frac{15}{35}$ is due to brakes $\Rightarrow$ 334.8 kJ

e) Typically rolling resistance = constant, wind resistance $\propto v^2$ so use formula like $R = k_1 + k_2 v^2$.

6.



Final velocities ~~&~~ v_p, v_q. → +ve

<u>Momentum conservation:</u>

$3m \times 2u - 2mu = 4mu$ (before)

$= 3mv_p + 2mv_q$ (after).

<u>Restitution:</u>

$$\frac{v_q - v_p}{3u} = e ,$$

Momentum ($\div m$) $\Rightarrow$ $3v_p + 2v_q = 4u$

$3v_q - 3v_p = 3\ (3eu)$

<u>add</u>:

$5v_q = (4+9e)u$

$v_q = \frac{1}{5}u(4+9e)$.

Then $v_p = \frac{4}{3}u - \frac{2}{3}v_q$

$= \frac{4}{3}u - \frac{2}{15}u(4+9e)$

$= u\left(\frac{4}{3} - \frac{8}{15} - \frac{18}{15}e\right)$.

so v_p negative if $\left(\frac{20-8}{15} - \frac{18e}{15}\right)$ is negative

i.e if $18e > 12$, $e > \frac{2}{3}$ so for $\underline{\frac{2}{3} < e < 1}$.

$e > \frac{4}{9}$.

6(c) Impulse on $Q = 2m(v_Q + u)$

$$= 2mu\left(\tfrac{1}{5}(4+9e) + 1\right)$$

$$= 2mu\left(\frac{9}{5} + \frac{9}{5}e\right)$$

$$= \frac{32}{5}mu.$$

$$\therefore \quad 2 \times \frac{9}{5}(1+e) = \frac{32}{5},$$

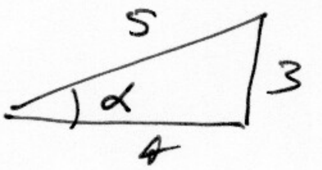
$$18(1+e) = 32, \quad 1+e = 1\tfrac{14}{18},$$

$$e = \frac{14}{18} = 7/9$$

7.

7.

$\nearrow V_s = 32 m/s$



$V_x = 32 \times 4/5 = 25.6 m/s$

$V_y = 32 \times 3/5 = 19.2 m/s$

a) Falls -20m, $u = 19.2 m/s$, $a = -9.8 m/s^2$

$\uparrow s_y$. $s_y = ut + \frac{1}{2}at^2$

$-20 = 19.2t - 4.9t^2$

$4.9t^2 - 19.2t - 20 = 0$

$t = \frac{19.2 \pm \sqrt{19.2^2 + 80 \times 4.9}}{9.8}$

$= 1.9592 \pm \frac{27.58}{9.8}$ sec

so (>0)

$t = 4.7734$ sec

b) $OC = 25.6 \times 4.7734 = 122.2 m$

c) $v^2 = u^2 + 2as$

$= 32^2 + 2 \times 9.8 \times 4 = 1102.4$,

$v = 33.2 m/s$

d)

$\cos\theta = \frac{25.6}{33.2}$,

$\theta = 39.55^\circ$