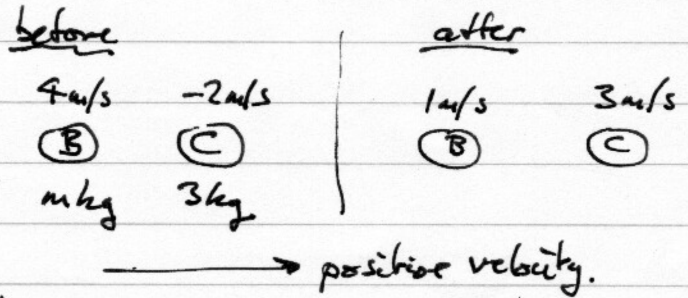
M1 Jan 2011

1.

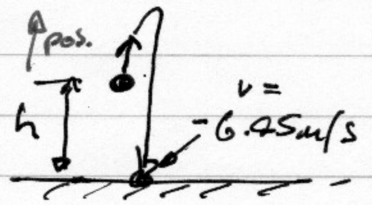


(a) "$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$", $4m + 3(-2) = 1m + 3\times 3$

$4m - 6 = m + 9$, $3m = 15$, $m = 5$ kg.

(b) "$Ft = mv - mu$", impulse $= m(v-u) = 3(3-(-2)) = 15$ Ns

2. (a)



$v = u + at$ (positive upwards ↑)

$-6.45 = u + (-9.8)\times 0.75$

$\therefore u = 9.8\times 0.75 - 6.45 = 0.9$ m/s

(b) At the highest point $v = 0$. $v^2 = u^2 + 2as$

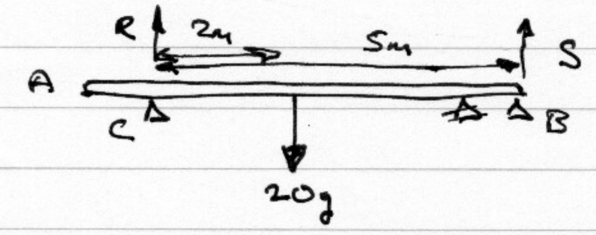
$0^2 = 0.9^2 + 2(-9.8)s$, $s = \dfrac{0.9^2}{2\times 9.8} = 0.0413$ m

(c) After falling h metres, $v = 6.45$ m/s

$v^2 = u^2 + 2as$, $s = \dfrac{v^2 - u^2}{2a} = \dfrac{6.45^2 - 0.9^2}{2(-9.8)} = -2.081$ m, $\therefore h = 2.08$ m

$\left[\text{or } \left(\frac{u+v}{2}\right)t = \left(\frac{6.45 - 0.9}{2}\right)\times 0.75\right]$.

3 (a)

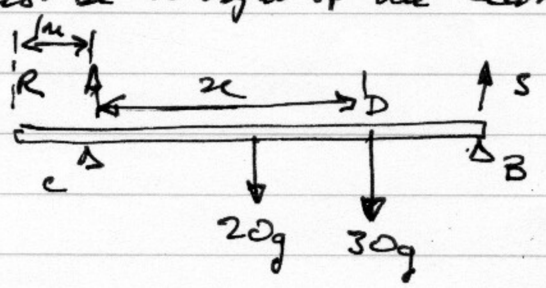


(at B).

Moments ↺ about C : $5S - 2\times 20g = 0$, $S = \frac{40}{5}g = 8g = 78.4$ N

Resolve vertically : $R + S - 20g = 0$, $R = 20g - 8g = 12g = 117.6$ N (at C).

(b) D must be to right of the centre. Now $R = S = \dfrac{20g + 30g}{2} = 25g$



Moments about C ↺ :

$2\times 20g + x(30g) - 5\times 25g = 0$

$(\div g)$: $40 + 30x - 125 = 0$

$30x = 125 - 40 = 85$

$\therefore AD = 1 + x = 1 + \frac{85}{30} = 3.833$ m

4. $\underline{u} = (2i - 5j)$ m/s at $t = 0$

$\underline{v} = (7i + 10j)$ m/s at $t = 5$

(a) Speed $= |\underline{u}| = \sqrt{2^2 + 5^2} = \sqrt{29} = 5.385$ m/s (5.39 to 3 s.fig).

(b) $\underline{F} = m\underline{a}$, $\underline{a} = \dfrac{\underline{v} - \underline{u}}{t} = \dfrac{(7i + 10j) - (2i - 5j)}{5} = \dfrac{5i + 15j}{5}$

$= (\underline{i} + 3\underline{j})$ m/s^2

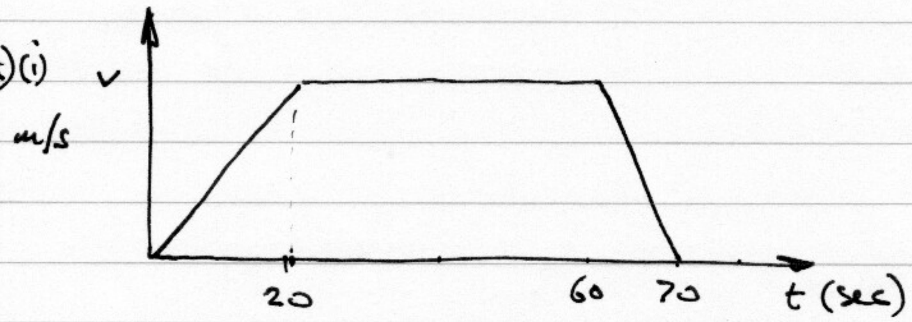
$\therefore \underline{F} = 2(i + 3j) = (2\underline{i} + 6\underline{j})$ N

(c) $\underline{v} = \underline{u} + \underline{a}t = (2i - 5j) + (i + 3j)t = (2 + t)\underline{i} + (-5 + 3t)\underline{j}$ m/s

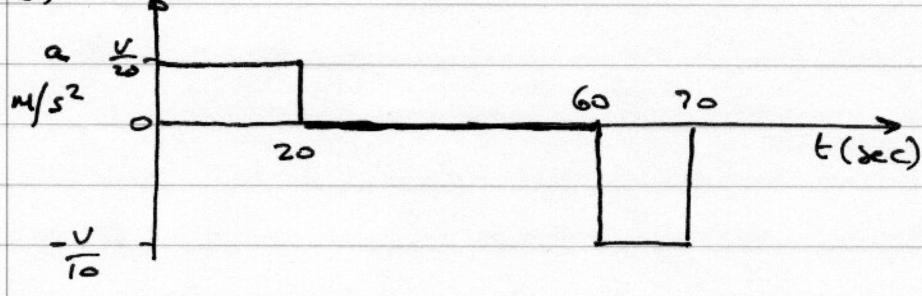
When $\underline{v}$ is parallel to $\underline{i}$ its $\underline{j}$ component is zero.

$\therefore -5 + 3t = 0$, $3t = 5$, $t = 5/3$ sec

5(a)(i)



(ii)

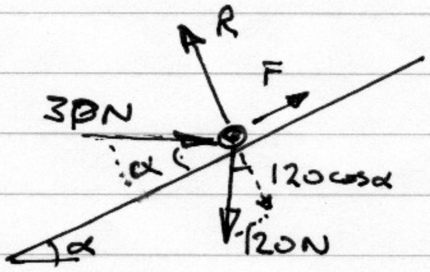


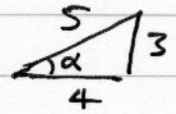
(b) Distance = area under speed-time graph

$= \tfrac{1}{2}(20v) + 40v + \tfrac{1}{2}(10v) = 55v = 880$ m

$\therefore v = \dfrac{880}{55} = 16$ m/s

6. (a)





$\sin\alpha = 3/5$

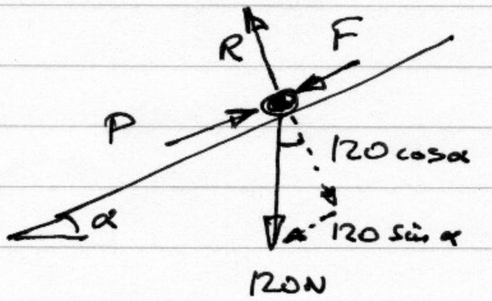
$\cos\alpha = 4/5$

Resolve perpendicular to slope ↑ so that F has no effect:

$R - 30\sin\alpha - 120\cos\alpha = 0$

$\therefore R = 30 \times 3/5 + 120 \times 4/5 = 114\,N$

(b)



The greatest value of P occurs when the weight is about to slide upwards.
The friction force F, trying to stop it sliding upwards, acts as shown.

Resolve ↑ $R - 120\cos\alpha = 0$, $R = 120 \times 4/5 = 96\,N$

Resolve ↗ $P - F - 120\sin\alpha = 0$

$F = \mu R = 1/2 \times 96 = 48\,N$ when about to slip.

$\therefore P = 48 + 120 \times 3/5 = 48 + 72 = 120\,N$

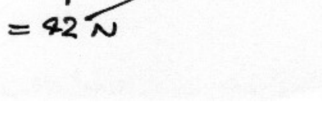
(c) $P - F - 120\sin\alpha = 0$ but now $P = 30\,N$

$F = P - 120\sin\alpha = 30 - 120 \times 3/5 = 30 - 72 = -42\,N$

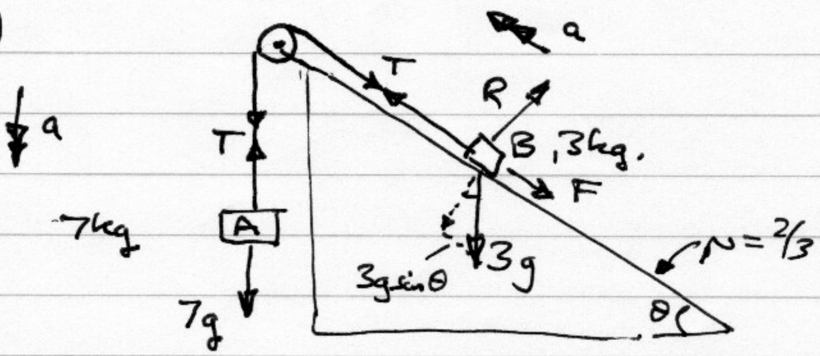
$\therefore$ the frictional force is $42\,N$

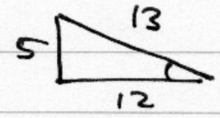
$F_{max} = \mu R = 48\,N$ so the particle is in equilibrium
[nb, if > 48, F = 48N and particle accelerates!].

F acts in the same direction as P (since was negative by above definition) ie up the slope.



7. (a)





$\sin\theta = 5/13$
$\cos\theta = 12/13$

<u>Equation of motion for A:</u> $(\Sigma F = ma)$

$7g - T = 7a$

<u>Equation of motion for B:</u>

$T - 3g\sin\theta - F = 3a$

<u>Add to eliminate T:</u>

$7g - 3g\sin\theta - F = 7a + 3a = 10a$

<u>Forces on B, resolving</u> ↗

$R - 3g\cos\theta = 0 \quad \therefore \quad R = 3g\cos\theta = 3g \times \frac{12}{13} = \frac{36}{13}g$

<u>Sliding</u> $\therefore \; F = \mu R = \frac{2}{3} \times \frac{36}{13}g = \frac{24}{13}g$

$10a = 7g - 3g\left(\frac{5}{13}\right) - \frac{24}{13}g = \left(7 - \frac{15}{13} - \frac{24}{13}\right)g = \frac{52}{13}g = 4g$

$\therefore a = \frac{4}{10}g = 3.92\,m/s^2$

(b) $v^2 = u^2 + 2as = 0^2 + 2 \times 3.92 \times 1 = 7.84, \; v = \sqrt{7.84} = 2.8\,m/s$

(c) String breaks, $T - 3g\sin\theta - F = 3a$ as before but now ~~F is~~ $T = 0$

~~negative~~ $3a = -3g(5/13) - \frac{24}{13}g = -\frac{39}{13}g = -3g$

$a = -g = -9.8\,m/s^2$

$v = u + at, \quad 0 = 2.8 - 9.8t, \quad t = \frac{2.8}{9.8} = 0.286$ sec.