# M1 January 2010

1. $\overrightarrow{\text{positive velocity.}}$

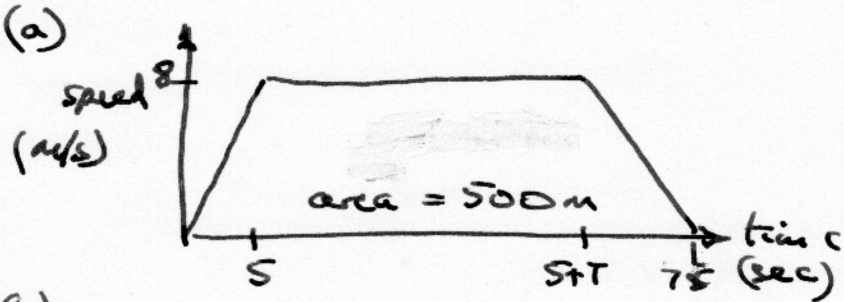
| | A | B |
|---|---|---|
| Initially | 12 m/s | −8 m/s |
| | 2 kg | m |
| Finally | 3 m/s | 4 m/s. |

(a) Impulse $= m(v-u) = 2(3-12) = -18$ Ns,
magnitude 18 Ns.

(b) $m(4-(-8)) = 18$ Ns $\quad \therefore \quad m = \frac{18}{12} = 1.5$ kg.

[or $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$,
$2 \times 12 - 8m = 2 \times 3 + 4m$, $\quad 24 - 6 = 4m + 8m$,
$18 = 12m$, $m = 1.5$].

2. (a)



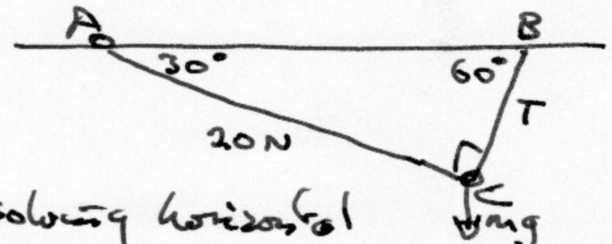

(b)

Area under line $= \frac{1}{2} \times 5 \times 8 + T \times 8 + (75 - (5+T))\, 8 \times \frac{1}{2} = 500$ m

$\therefore \ 20 + 8T + 4(70 - T) = 500$

$300 + 4T = 500, \quad 4T = 200, \quad T = 50$ sec.

3.




(a) Resolving horizontal forces on the particle, $20\cos 30 - T\cos 60 = 0$

$\therefore \ T = \frac{20\cos 30}{\cos 60} = \frac{20\cos 30}{\sin 30} = \frac{20}{\tan 30} = 34.64$ N or $20\sqrt{3}$ N.

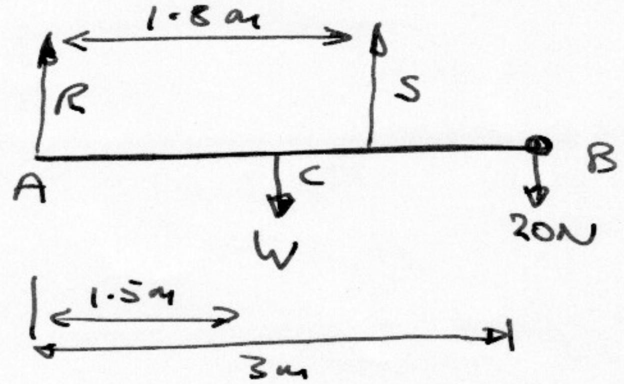
(b) Resolving vertical forces,

$mg - 20\sin 30 - 20\sqrt{3}\sin 60 = 0$

$mg = 20 \times \frac{1}{2} + 20\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 10 + 30 = 40$N

$m = 40 \div 9.8 = 4.08$ kg.

4.




(a) Moments about A: $1.8S - 1.5W - 3\times 20 = 0$

$$S = \frac{1.5W + 60}{1.8} = \frac{15W + 600}{18} = \frac{5W + 200}{6}$$
$$= \left(\frac{5}{6}W + \frac{100}{3}\right) N$$

(b) Resolving vertically,

$R + S - W - 20 = 0 \quad \therefore R = W + 20 - S$

$$R = 20 - \frac{100}{3} + (1 - 5/6)W = \left(\frac{-40}{3} + \frac{1}{6}W\right) N$$

(c) $S = 8R$

$$\frac{5}{6}W + \frac{100}{3} = 8\left(\frac{-40}{3} + \frac{1}{6}W\right)$$

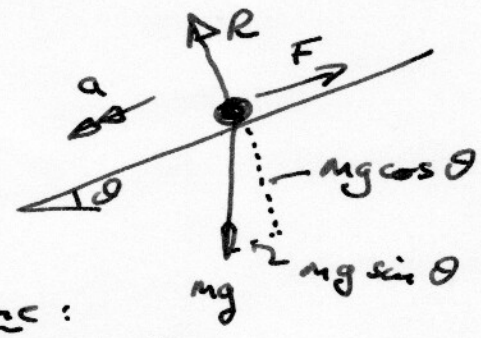
$(\times 6)$: $5W + 200 = 8(-80 + W) = -640 + 8W$

$840 = 3W, \quad W = 280\,N$

5. (a) 2.7m in 3 sec, from rest. $s = ut + \frac{1}{2}at^2$

$2.7 = \frac{1}{2}at^2, = \frac{9}{2}a \quad \therefore a = \frac{5.4}{9} = 0.6 m/s^2$

(b)




$m = 0.8kg$
$\theta = 30°$

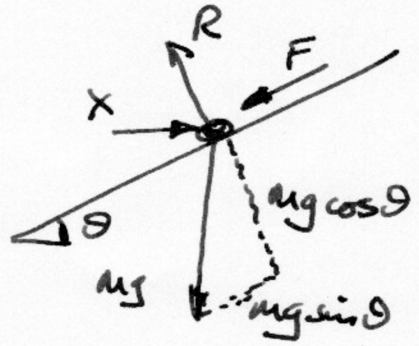
Down plane:

$mg\sin\theta - F = ma, \quad F = m(g\sin\theta - a) = 0.8(9.8\times\frac{1}{2} - 0.6)$
$= 3.44N$

$\perp$ to plane: $R - mg\cos\theta = 0, \quad R = 0.8\times 9.8\cos 30 = 6.79N$

Sliding, $\therefore \mu = F/R = 0.5067 \approx 0.51$.

5(c)




F downwards as on point of moving up.
X changes both R and F
⇒ need simultaneous equations.

Need to resolve in two different directions. ↑ and → are easiest as → equation gives X directly and ↑ equation lets us find R, F before we know X.

Resolve ↑: $R\cos\theta - F\sin\theta - mg = 0$

Limiting equilibrium $\therefore F = \mu R = 0.5067R$

$\therefore R(\cos 30 - 0.5067\sin 30) = 0.8\times 9.8$

$$R = \frac{0.8\times 9.8}{\cos 30 - 0.5067\sin 30} = 12.796\text{ N},$$

$F = 0.5067R = 6.484\text{ N}.$

Resolve →

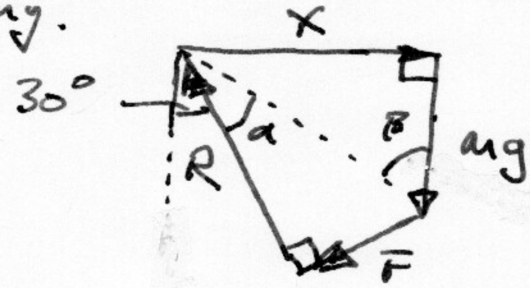
$X = R\sin\theta + F\cos\theta$

$= 12.796\sin 30 + 6.484\cos 30 = 12.01\text{ N}$

$\approx 12\text{ N}.$

Alternatively

Draw a force diagram. Must draw R & F together as we know the angle between them, similarly X and mg.




$\tan\alpha = F/R = \mu = 0.5067$

$\therefore \alpha = \tan^{-1}(0.5067) = 26.87°$

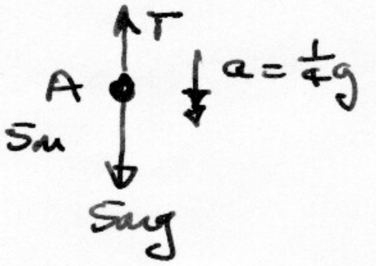
$\beta = 30 + 26.87 = 56.87°$

$X = mg\tan\beta$

$= 0.8\times 9.8\tan 56.87$

$= 12.01\text{ N}$

6 (a)

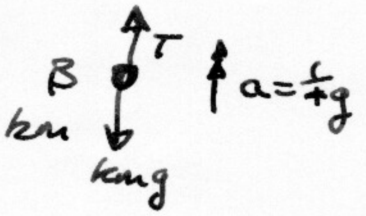



$5mg - T = 5m(\frac{1}{4}g)$

$5m(g - \frac{1}{4}g) = T$

$T = 5m(3/4\,g) = \frac{15}{4}mg$

(b)




$T - kmg = km(\frac{1}{4}g)$

$\frac{15}{4}mg = kmg + \frac{1}{4}kmg = \frac{5}{4}kmg$.

$15 = 5k$, $k = 3$.

(c) Smooth → no friction, T is same each side of the pulley.

(d) When A hits floor: $s = ut + \frac{1}{2}at^2 = \frac{1}{2}(\frac{1}{4}g)1.2^2 = 1.764$m

At this instant, $v = u + at = \frac{1}{4}g \times 1.2 = 2.94$ m/s

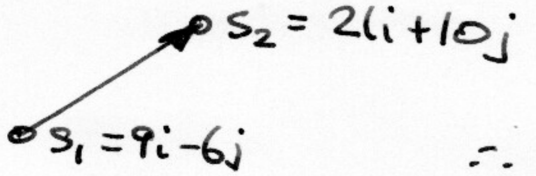
Flying upwards, $v^2 = u^2 + 2as$, $0 = 2.94^2 + 2(-9.8)s$,

$s = 0.441$m.

Total height $1.764 + 0.441 = 2.205$m

---

7(a)



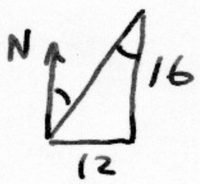

$s_2 - s_1 = (21i + 10j) - (9i - 6j)$

$= 12i + 16j$ in 4 hours

$\therefore \underline{v} = (3i + 4j)$ km/h

Speed $= \sqrt{3^2 + 4^2} = 5$ km/h

(b)




Bearing $= \tan^{-1}(12/16) = 36.87° = 036.9°$

(c) $s = (9i - 6j) + (3i + 4j)t = (9 + 3t)i + (4t - 6)j$

(d) $\vec{LS} = (9 + 3t)i + (4t - 6)j - (18i + 6j) = (3t - 9)i + (4t - 12)j$

$|LS| = 10 \therefore (3t - 9)^2 + (4t - 12)^2 = 10^2$

$9t^2 - 54t + 81 + 16t^2 - 96t + 144 = 100$

$25t^2 - 150t + 125 = 0$, $t^2 - 6t + 5 = 0$

$(t - 1)(t - 5) = 0$, $T = 1$ hour, 5 hours.