M1 Jan 2008

1. **Before** **After**

$5\,m/s$ $-3\,m/s$ $1\,m/s$ v_b

A B A B

$4\,kg$ m

$\rightarrow$ +ve vel.

(a) ~~Momentum~~

Impulse = change of momentum $= mv - mu$

$$= 4(1-5) = -16\ Ns$$

$\therefore$ Magnitude $= 16\ Ns$

(b) Momentum conservation

$$m_a u_a + m_b u_b = m_a v_a + m_b v_b$$

$$4 \times 5 + (-3m) = 4 \times 1 + \cancel{m v_b}\ 2m$$

$$4(5-1) = 16 = m(2+3)$$

$v_b = \underline{\underline{+2}}$ since A is moving to right, so B must be too.

$$m = \frac{16}{5} = 3.2\ kg$$

[or Impulse on B $= 16\ Ns = m(2-(-3)) = 5m$]

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2 (a) $s = ut + \frac{1}{2}at^2$, $u = 0$ ("from rest")

$$27 = \tfrac{1}{2}a(3^2), \quad a = 2 \times \frac{27}{9} = 6\,m/s^2$$

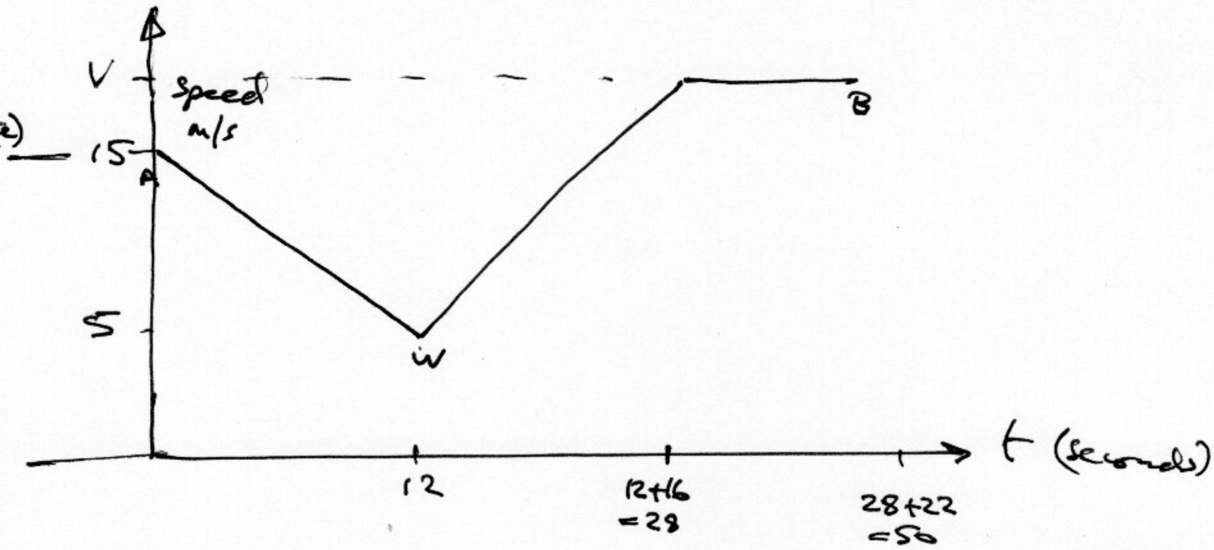
(b) $v = u + at = 0 + 6 \times 3 = 18\,m/s$

(c) 2 seconds after burn-out:

$$h = 27 + ut + \tfrac{1}{2}at^2 = 27 + 18 \times 2 - \tfrac{1}{2} \times 9.8 \times 2^2$$

$$= 43.4\,m$$

3. (a)



Braking:

$v^2 = u^2 + 2as$, $\quad 5^2 = 15^2 + 2a \times 120$,

$a = -5/6 \ m/s^2$

$v = u + at$, $\quad 5 = 15 - 5/6\, t$, $\ t = 12$ seconds to W.

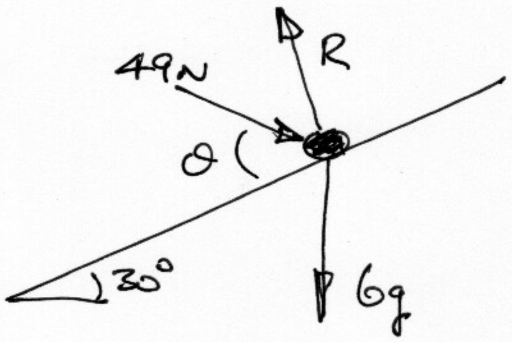
(b) Total time $12 + 16 + 22 = 50$ seconds

(c) $1000m = \left(\frac{15+5}{2}\right)12 + \left(\frac{\cancel{15}\,5 + V}{2}\right)16 + 22V$

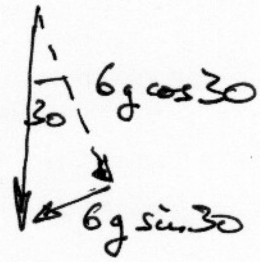
$= 120 + \cancel{96}\,40 + 30V$

$V = \frac{\cancel{784}\,840}{30} = \cancel{26\tfrac{2}{15}\,m/s}\ 28\ m/s$

4.



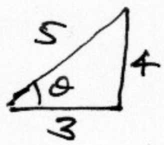
Components of 6g:



a) Resolve parallel to the slope (to get a $\cos\theta$ term, which we want to find, and avoid using R, which we don't know):

$$49\cos\theta - 6g\sin 30 = 0$$

$$\cos\theta = \frac{6\times 9.8\times \frac{1}{2}}{49} = 3/5$$



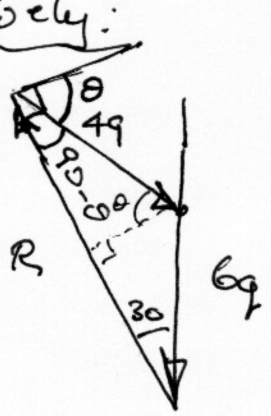
b) Resolve ⊥ to plane (or any direction with a non-zero R-component)

$$R - 49\sin\theta - 6g\cos 30 = 0$$

$$R = 49\times(4/5) + 6\times 9.8\cos 30 = 90.1\text{ N}$$

Nb questions involving "g" must be answered to 2 or 3 significant figures, you will lose a mark for 1 or 4 s. figs.

Alternatively:



Sine rule: $\frac{\sin(90-\theta)}{6g} = \frac{\sin 30}{49}$

$$\sin(90-\theta) = \frac{6\times 9.8\times\frac{1}{2}}{49} = 3/5 = \cos\theta$$

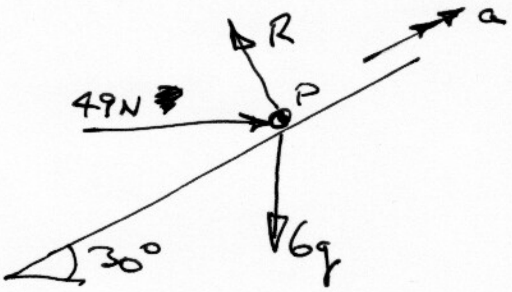
$\Rightarrow \theta = 53.13^\circ$

$180-30-(90-53.13) = 113.13^\circ$

$R^2 = (6g)^2 + 49^2 - 2(6g)49\cos 113.13,$

$R = 90.1\text{ N}$

4(c)



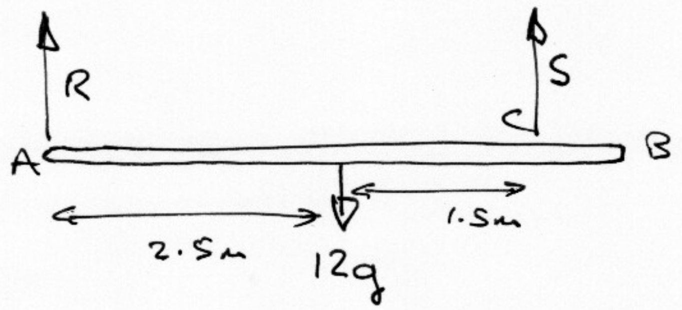
Resolve up the plane:

$$49\cos 30 - 6g\sin 30 = ma = 6a$$

$= 13.0\,N$

$\therefore\ a = 2.17\,m/s^2$

5. (a)



Take moments about A (so R is not in the equation, we find S directly):

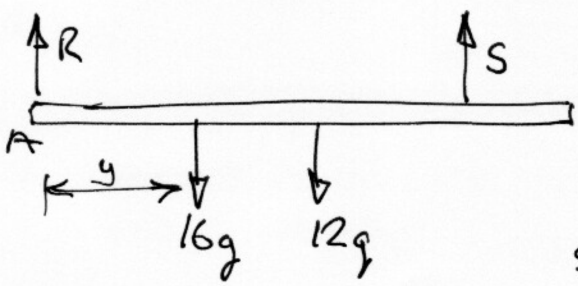
$$2.5 \times 12g - 4S = 0$$

$$S = 7.5g = 73.5N \quad \text{(to 3 s. figs)}.$$

(b) $R + S - 12g = 0,$

$$R = 12g - 7.5g = 4.5g = 44.1\,N$$

(c)



Moments about A:

$$16gy + 2.5\times 12g - 4S = 0$$

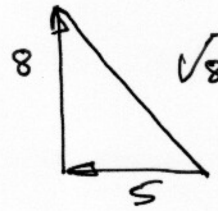
$$S = (7.5 + 4y)g$$

$$= (73.5 + 39.2y)\,N$$

If $S = 98N$, $y = 5/8\,m$ so safe for $0 \leq y \leq 5/8\,m$.

6.

a)



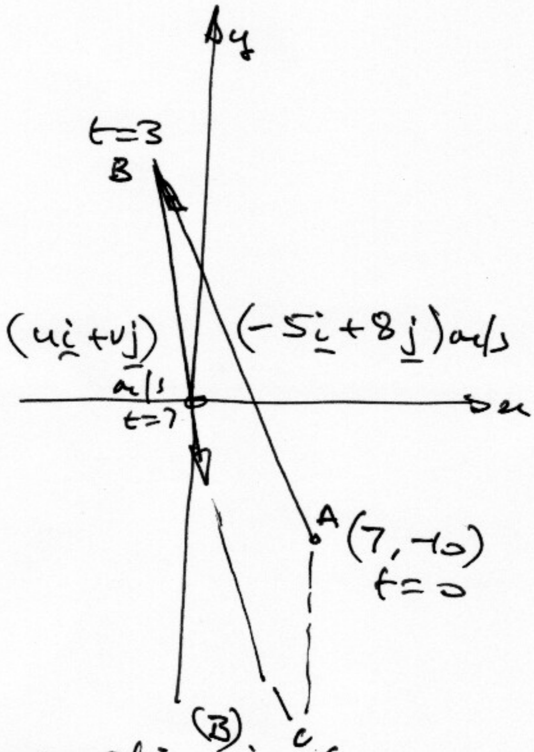
$\sqrt{8^2+5^2} = \sqrt{89} = 9.43$ m/s

b)

$\tan\theta = 8/5 = 1.6$, $\theta = 58°$,

bearing = 270 + 58 = 328°

(c)



At $t=3$, position is $(7i-10j) + 3(-5i+8j) = (7-15)i + (24-10)j$
$= (-8\underline{i} + 14\underline{j})$ m

∴ Moves $(8\underline{i} - 14\underline{j})$ to reach the origin O, in 4 seconds.

$u\underline{i} + v\underline{j} = \frac{1}{4}(8\underline{i} - 14\underline{j}) = (2\underline{i} - 3\tfrac{1}{2}\underline{j})$ m/s,

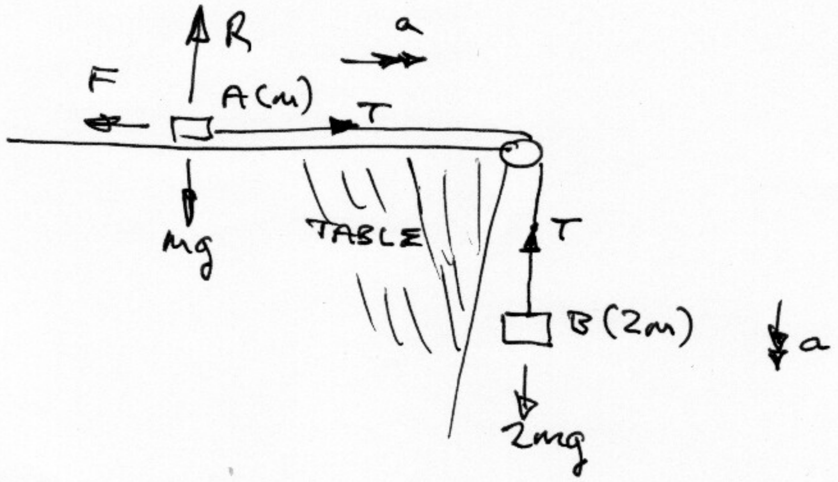
$u = 2$, $v = -3\tfrac{1}{2}$.

(d) Moving from O to point C with i-component = 7 at 2m/s in the i-direction will take $7/2 = 3\tfrac{1}{2}$ seconds.

∴ Total time = $3 + 4 + 3\tfrac{1}{2} = 10\tfrac{1}{2}$ seconds

Jan 08 M1

7.



(a) <u>Motion of B</u> (so we can find T without knowing μ):

$2mg - T = 2ma$

$a = 4/9\,g$, so

$T = 2mg - 2m(4/9\,g) = 2m(5/9\,g) = \frac{10}{9}mg$ N

$= 10.9m$ N

(b) <u>Motion of A</u>:

$T - F = ma = 4/9\,mg$

$F = T - 4/9\,mg = (10/9 - 4/9)mg = 2/3\,mg$

Resolving vertically at A, $R - mg = 0$, $R = mg$.

Then (since sliding) $\mu = F/R = 2/3$

(c) $v^2 = u^2 + 2as = 0 + 2 \times (4/9\,g) \times h$,

$V = (2/3)\sqrt{2gh}$ when B hits the ground.

After that, F is still $2/3\,mg$ but $T = 0$

$T - F = ma$ as before for A, $a = -2/3\,g$

Now put $u = (2/3)\sqrt{2gh}$, want v after a further $s = h/3$:

$v^2 = u^2 + 2as = (8/9)gh + 2(-\frac{2}{3}g)(h/3) = \frac{4}{9}gh$, so

speed of A at P $= \sqrt{\frac{4gh}{9}} = 2/3\sqrt{gh}$

(d) The lightness of the string means that it will have constant tension T, even when accelerating, at all points along it.