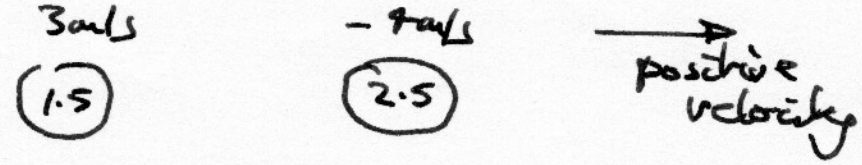
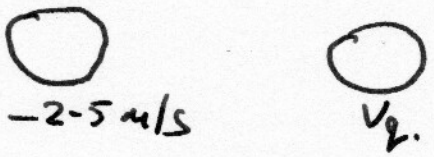
M1 Jan 2005

1(a) Before



After



$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$1.5\times 3 + 2.5(-4) = 1.5(-2.5) + 2.5v_2$

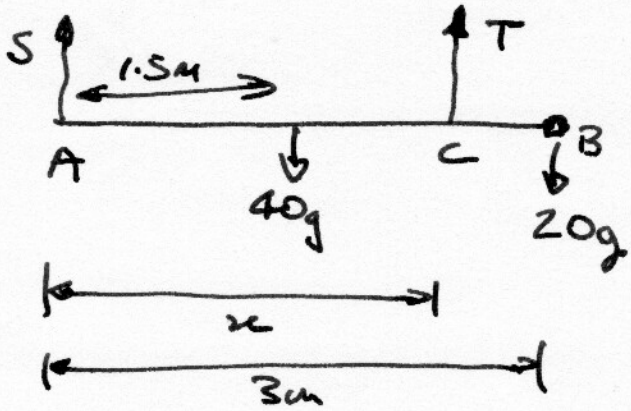
$\therefore 2.5v_2 = -1.75, \quad v_2 = -0.7$ m/s

(b) Negative velocity, as before → has not changed.

(c) Impulse $= |m_1(v_1 - u_1)| = |1.5(-2.5-3)| = 8.25$ Ns

(magnitude, don't need − sign).

2.



(a) $T = 3S$

$S + T = 40g + 20g = 60g$

$= 4S$

$\therefore S = 15g, \quad T = 45g$

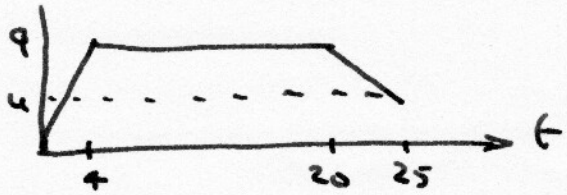
(b) Moments about A:

$40\times 1.5 + 20\times 3 - 45x = 0$ (leaving out factor g).

$45x = 120, \quad x = \frac{120}{45} = \frac{8}{3}$ m $\therefore$ Distance $CB = 3 - \frac{8}{3} = \frac{1}{3}$ m

[check: $\frac{8}{3} < 3$ m ✓].

3.



(a) Area up to $t = 20$

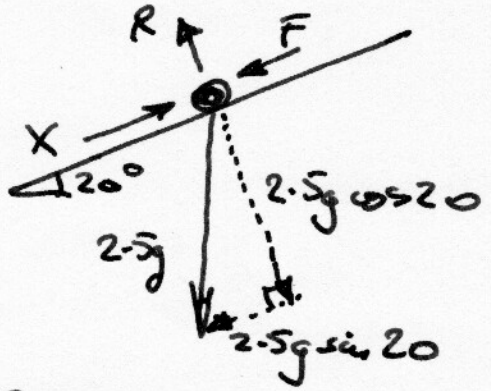
$\frac{1}{2}(4 \times 9) + 9(20 - 4) = 18 + 144 = 162$ m

(b) Area between 20 & 25 sec $= 200 - 162 = 38$

$\left(\frac{9+u}{2}\right)5 = 38$, $\quad 9 + u = \frac{76}{5} = 15.2 \quad \therefore u = 6.2$ m/s

(c) Deceleration $= \frac{9 - 6.2}{5} = \frac{2.8}{5} = 0.56$ m/s^2

4.



"about to move up" so F acts down the slope.

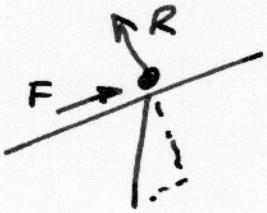
(a) $R - 2.5g \cos 20 = 0$, $\quad R = 2.5g \cos 20 = 23.02$ N

(b) $\mu = 0.4$, limiting equilibrium so $F = \mu R$

$\therefore F = 9.209$ N, $\quad X - F - 2.5g \sin 20 = 0$

$\therefore X = 11.59$ N

(c) Now



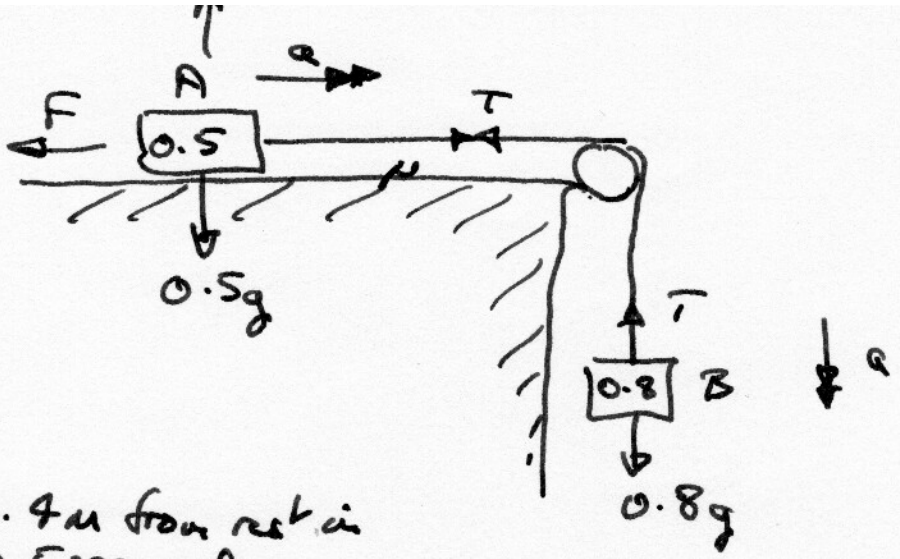
, $R = 23.02$ N as before

$F = 2.5g \sin 20 = 8.38$ N

$\frac{F}{R} = \frac{8.38}{23.02} = 0.364$, < 0.4 so does not slide.

[or $\tan 20° = 0.364$, " " "]

5.



(a) 0.4m from rest in 0.5 seconds

$s = ut + \frac{1}{2}at^2$, $0.4 = 0 + \frac{1}{2}a(0.5)^2$

$\therefore a = 0.4 \times 8 = 3.2 \text{ m/s}^2 = \frac{a}{8}$

(b) "$F = ma$" for B:

$0.8g - T = 0.8 \times 3.2$

$T = 0.8(g - 3.2) = 5.28 \text{ N}$

(c) "$F = ma$" for A:

$T - F = 0.5 \times 3.2 = 1.6 \text{ N}$

$\therefore F = T - 1.6 = 3.68 \text{ N}$

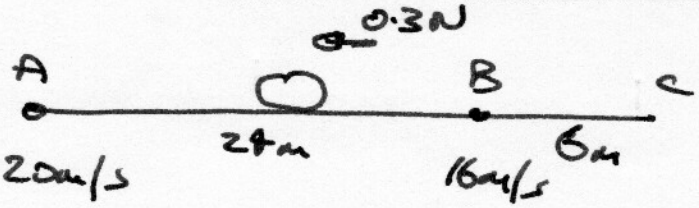
$R = 0.5g$.

$\mu = \frac{F}{R}$ (since in motion)

$= \frac{3.68}{4.9} = 0.751$

(d) "Inextensible" means both masses always have the same speed, so their accelerations are equal.

6.



$v^2 - u^2 = 2as$, $16^2 - 20^2 = 2a(24)$

$\therefore a = -3 \text{ m/s}^2$, deceleration 3 m/s^2

6(b) $v^2 = u^2 + 2as, = 16^2 + 2(-3)6 = 220,$

$v = \sqrt{220} = 14.83 \text{ m/s}$

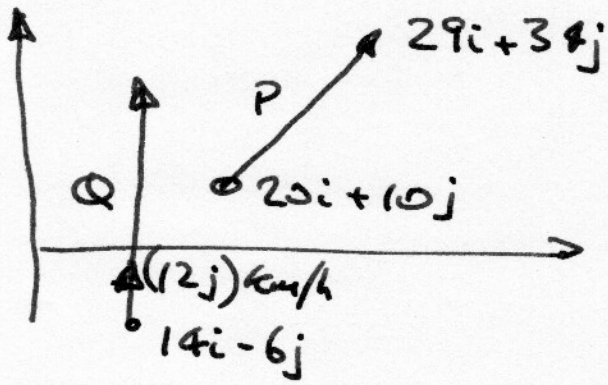
(c) $F = ma, \quad 0.3 = 3m \quad \therefore m = 0.1 \text{ kg}$

(d) Impulse $= m(v-u), \quad |v-u| = \frac{2.4 \text{ Ns}}{0.1 \text{ kg}} = 24 \text{ m/s}$

$v = 14.83 - 24 = -9.17 \text{ m/s}$

$t = \frac{9.17}{3 \text{ m/s}^2} = 3.06 \text{ sec}$

7.



(a) $v = \frac{(29i + 34j) - (20i + 10j)}{3} = \frac{9i + 24j}{3} = (3i + 8j) \text{ km/h}$

(b) $p = p_0 + vt = (20i + 10j) + (3i + 8j)t$

$= [(20 + 3t)i + (10 + 8t)j] \text{ km}$

$q = (14i - 6j) + 12jt = [14i + (12t - 6)j] \text{ km}$

(c) $\overrightarrow{PQ} = q - p = (-6 - 3t)i + (-16 + 4t)j$

$d^2 = |PQ|^2 = (-6 - 3t)^2 + (-16 + 4t)^2$

$= 36 + 36t + 9t^2 + 256 - 128t + 16t^2$

$= 25t^2 - 92t + 292$

(d) At $t = 1$, $d^2 = 25 - 92 + 292 = 225 \therefore d = 15$

Solve for second root at $d^2 = 225$:

$25t^2 - 92t + 292 = 225, \quad 25t^2 - 92t + 67 = 0$

$ac = 25 \times 67 = -25 \times -67$, adds to -92

$(25t - \frac{67}{1})(t - \frac{25}{25}) = (25t - 67)(t - 1) = 0$

$\therefore t = \frac{67}{25} = 2.68 \text{ hours} = 2\text{h } 40.8 \text{ min}; \ 2{:}41 \text{ am}.$