C3 JANUARY 2009

1. (a) $y = x^2(5x-1)^{1/2}$, need product rule

$dy/dx = u'v + uv'$

$= 2x(5x-1)^{1/2} + x^2\left[\frac{1}{2}(5x-1)^{-1/2} \times 5\right]$

At $x=2$, $dy/dx = 4\sqrt{9} + 4\left[\frac{5}{2}\ \frac{1}{\sqrt{9}}\right]$

$= 12 + \frac{10}{3} = 15\frac{1}{3}$

(b) Quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

$$\frac{d}{dx}\left(\frac{\sin 2x}{x^2}\right) = \frac{(2\cos 2x)x^2 - (\sin 2x)2x}{x^4}$$

$$= \frac{2x\cos 2x - 2\sin 2x}{x^3}$$

2.

(a) $f(x) = \frac{2\cancel{(x+1)}}{(x-3)\cancel{(x+1)}} - \frac{(x+1)}{(x-3)} = \frac{1-x}{x-3}$

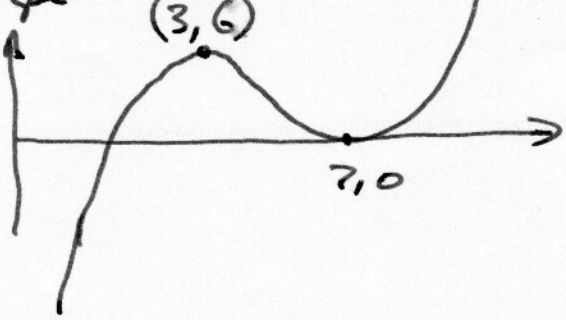
(b) $f'(x) = \frac{u'v - uv'}{v^2} = \frac{-(x-3) - (1-x)(1)}{(x-3)^2} = \frac{-x+3+x-1}{(x-3)^2}$

$= \frac{2}{(x-3)^2}$

---

3. (a) BIDMAS! stretch $\times 2 \uparrow$, move down 4

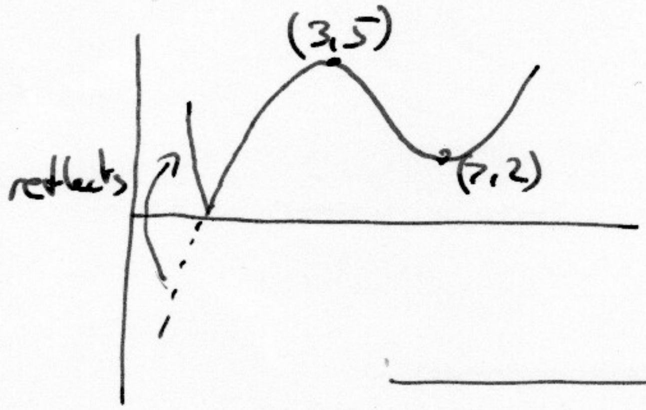
Same shape



$2 \times 5 - 4 = 6$

$2 \times 2 - 4 = 0$

3(b) $y = |f(x)|$, any negative y values become positive.



4. $x = \cos(2y+\pi)$, $\quad dx/dy = -2\sin(2y+\pi)$

At $y = \pi/4$, $\quad dx/dy = -2\sin(\pi/2 + \pi) = -2(-1) = 2$

$\therefore \; dy/dx = \frac{1}{2} \quad \left(= \frac{1}{(dx/dy)}\right).$

$y - \pi/4 = \frac{1}{2}(x-0)$, $\quad y = \frac{1}{2}x + \pi/4$

5. (a) $f(x) \equiv 3x + \ln x$, $\quad g(x) \equiv e^{x^2}$

$x^2 \geq 0$, $\; e^{x^2} \geq 1 \quad \therefore g(x) \geq 1$

(b) $f(g(x)) = 3e^{x^2} + \ln(e^{x^2}) = 3e^{x^2} + x^2$

(c) $g(x) \geq 1$, $\ln(1) = 0$ so $f(g(x)) \geq 3$

(d) $\frac{d}{dx} fg(x) = 6xe^{x^2} + 2x$

$\therefore \; 6xe^{x^2} + \cancel{2x} = x^2e^{x^2} + \cancel{2x}$

$(x^2 - 6x)e^{x^2} = 0 \qquad e^{x^2} \neq 0$ for real x

$\therefore \; x(x-6) = 0$, $\quad x = 0$ or 6

but $x > 0 \quad \therefore \; x = 6$

6(a)(i) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$

$= (2\sin\theta\cos\theta)\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$

$\Rightarrow$ We want sines only, so use $\cos^2\theta = 1-\sin^2\theta$

$\sin 3\theta = \cos^2\theta\left[2\sin\theta + \sin\theta\right] - \sin^3\theta$

$= (1-\sin^2\theta)\left[3\sin\theta\right] - \sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

(ii)

$8\sin^3\theta - 6\sin\theta = -2(3\sin\theta - 4\sin^3\theta) = -2\sin 3\theta$

$\therefore\ 8\sin^3\theta - 6\sin\theta + 1 = -2\sin 3\theta + 1 = 0$

$\therefore\ 2\sin 3\theta = 1, \quad \sin 3\theta = 1/2$

$\sin 3\theta = \pi/6,\ \pi - \pi/6 \ + n2\pi$

$= \pi/6,\ 5\pi/6 \ + n2\pi$

$\therefore\ \theta = \pi/18,\ \frac{5\pi}{18} + n\frac{2}{3}\pi$

$= \frac{\pi}{18},\ \frac{5\pi}{18}$ in range $0 < \theta < \pi/3$

(b) $\sin 15° = \sin(45-30) = \sin 45\cos 30 - \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) = \frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$

---

7. $f(x) = 3xe^x - 1$

(a) At turning point gradient $f'(x) = 0$

$f'(x) = 3e^x + 3xe^x = 3(x+1)e^x$

$e^x \neq 0 \ \therefore\ x+1 = 0,\ x = -1,\ y = 3(-1)e^{-1} - 1$

so P is $(-1, -3e^{-1} - 1)$.

7(b) Calculator 0.25=

$e^{\square}(-\text{ANS}) \div 3 = 0.2596,$

(repeating =) $0.2571,$

$0.2578.$

(c) $f(x)$ is continuous (since e^x is continuous).

$f(0.25755) = -3.79 \times 10^{-4}$
$f(0.25765) = 1.09 \times 10^{-4}$ } sign change in $f(x)$ so there is a root that rounds to 0.2576.

8. (a) $R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

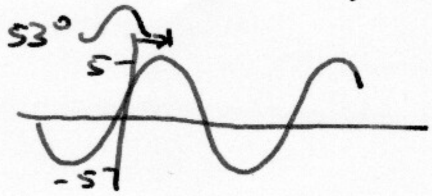
$= (R\cos\alpha)\cos\theta + (R\sin\alpha)\sin\theta$

$= 3\cos\theta + 4\sin\theta$

$\therefore R\cos\alpha = 3, \quad R\sin\alpha = 4$

$R = \sqrt{3^2+4^2} = 5, \quad \tan\alpha = 4/3, \quad \alpha = 53.13°$

(b)



$\cos(\theta - 53.13)$, $\cos\theta$ shifted $\rightarrow 53.13°$

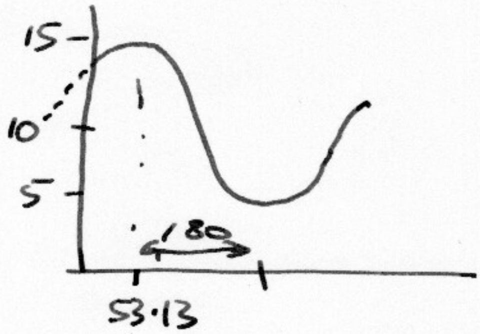
so max value 5, at $\theta = 53.13°$.

(c) $f(t) = 10 + 3\cos 15t + 4\sin 15t$

$= 10 + 5\cos(\theta - 53.13), \quad \theta = 15t$

Min. temperature $10 - 5 = 5$

(d)



$\theta = 180 + 53.13 = 233.13°$

$t = \theta/15 = 15.54$ hours after midday.