C2 JUNE 2010

1.(a) Use calculator ANS button.

$0 =$

$3\boxed{x^{\square}}$ANS$+2$ANS$=$, gives 1 ✓ (check with given value).

Then $0.6 =$, ▲ to get formula back, $=$ etc

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65	2.35	3.13	4.01	5

[Can use Table mode, but this is probably easier].

(b) $h = 0.2 - 0 = 0.2$ or $h = \frac{b-a}{n} = \frac{1-0}{5} = 0.2$

$\int_a^b y\,dx \approx \frac{h}{2}\left(y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4)\right)$

$= \frac{0.2}{2}\left(1 + 5 + 2(1.65 + 2.35 + 3.13 + 4.01)\right)$

$= 2.828$

2. $f(x) = 3x^3 - 5x^2 - 58x + 40$

(a) Let $x - 3 = x - a \Rightarrow a = 3$

$f(a) = f(3) = 3(27) - 5(9) - 58(3) + 40 = -98 =$ remainder

(b)
$$\begin{array}{r} 3x^2 + 10x - 8 \\ x-5\,\overline{)\,3x^3 - 5x^2 - 58x + 40} \\ \underline{3x^3 - 15x^2} \quad\quad\quad\;\; - \\ 10x^2 - 58x + 40 \\ \underline{10x^2 - 50x} \quad\;\; - \\ -8x + 40 \\ \underline{-8x + 40} \; - \\ 0 \end{array}$$

$ac = 3 \times -8 = -24$

$= 12 \times -2$

$\therefore 3x^2 + 10x - 8$

$= \left(3x - \frac{2}{1}\right)\left(1x + \frac{12}{3}\right)$

$= (3x - 2)(x + 4)$

$f(x) = (x-5)(3x-2)(x+4)$

$f(x) = 0$ has solutions $x = -4, \frac{2}{3}, 5$.

3(a) $y = x^2 - k\sqrt{x} = x^2 - kx^{1/2}$

$dy/dx = 2x - \frac{1}{2}kx^{-1/2}$

3(b) At $x = 4$, $dy/dx = 8 - \frac{\frac{1}{2}k}{\sqrt{4}} = 8 - \frac{1}{4}k$

If y is decreasing, $dy/dx < 0$ $\therefore$ $8 - \frac{1}{4}k < 0$

$8 < \frac{1}{4}k$, $32 < k$, $k > 32$

4(a) $(1+ax)^7 = 1 + {}^7C_1(ax) + {}^7C_2(ax)^2 + {}^7C_3(ax)^3 + \dots$

$= 1 + 7ax + 21a^2x^2 + 35a^3x^3 + \dots$

(b) $21a^2 = 525$

$\therefore a^2 = \frac{525}{21} = 25$, $a = \pm 5$

5(a) $5\sin\theta = 2\cos\theta$, $\sin\theta = \frac{2}{5}\cos\theta$,

$\tan\theta = \frac{\sin\theta}{\cos\theta} = 2/5$

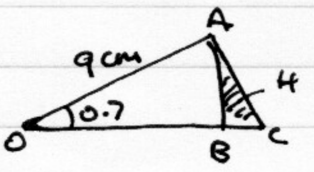
(b) Let $2x = \theta$, $\tan\theta = 2/5$, $\theta = \tan^{-1}(2/5)$ & repeats

$= 21.8^\circ + n180^\circ$

$\therefore x = \frac{\theta}{2} = 10.9^\circ + n90^\circ$

for $0 < x < 360$, $x = 10.9^\circ, 100.9^\circ, 190.9^\circ, 280.9^\circ$.

6(a)



Arc length $= r\theta$

$\therefore$ length of arc $AB = 9 \times 0.7 = 6.3$ cm

(b) Sector area $= \frac{1}{2}r^2\theta = \frac{1}{2}(9^2)0.7 = 28.35$ cm^2

(c) $\frac{AC}{OA} = \tan(0.7)$, $AC = 9\tan(0.7) = 7.5806$ cm $= 7.58$ cm (2dp.)

(d) H area = triangle area − sector area

$= \frac{1}{2} \times 9 \times 7 - 28.35 = 3.15$ cm^2

7(a) $2\log_3(x-5) - \log_3(2x-13) = 1$

$= \log_3(x-5)^2 - \log_3(2x-13) = \log_3\left(\frac{(x-5)^2}{2x-13}\right)$

$3^{LHS} = 3^{RHS} \Rightarrow \frac{x^2 - 10x + 25}{2x-13} = 3^1 = 3$

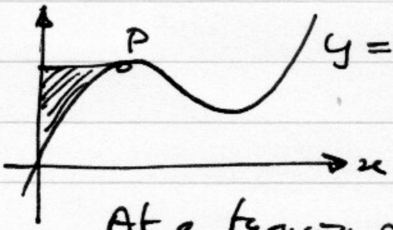
$\therefore x^2 - 10x + 25 = 3(2x - 13) = 6x - 39$

$x^2 - 16x + 64 = 0$

(b) $x^2 - 16x + 64 = (x-8)(x-8) = 0$

$\therefore x = 8$

8(a)



$y = x^3 - 10x^2 + kx$

$dy/dx = 3x^2 - 20x + k$

At a turning point, $dy/dx = 0$

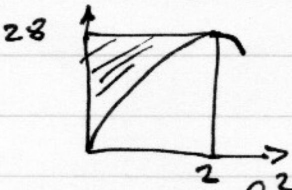
The turning point is at $x = 2$

$\therefore 3(2^2) - 20 \times 2 + k = 0, \quad -28 + k = 0,$

$k = 28$

(b) At P, $y = 2^3 - 10(2^2) + 28 \times 2 = 24$

Area of R = area of rectangle − area under curve



Area under curve $= \int_0^2 x^3 - 10x^2 + 28x \, dx$

$= \left[\frac{x^4}{4} - \frac{10}{3}x^3 + 14x^2\right]_0^2 = \left(\frac{16}{4} - \frac{80}{3} + 56\right) - (0) = \frac{100}{3}$

$\therefore$ Area of $R = 28 \times 2 - \frac{100}{3} = 22\frac{2}{3}$

9(a) Geometric sequence (not series!) $u_n = ar^{n-1}$

End of year two (after 1 year) population $= 25000 \times 1.03$

$= 25750$

(b) Common ratio $r = 1.03$

(c) $ar^{n-1} > 40000, \quad 25000 \times 1.03^{n-1} > 40000$

$1.03^{n-1} > \frac{40000}{25000} \quad \therefore 1.03^{n-1} > 1.6, \quad (n-1)\log 1.03 > \log 1.6$

(d) $(N-1)\log 1.03 > \log 1.6 \quad \therefore \quad N-1 > \dfrac{\log 1.6}{\log 1.03}$

$\therefore N-1 > 15.9, \qquad N > 16.9$

$\therefore N = 17$, the seventeenth year.

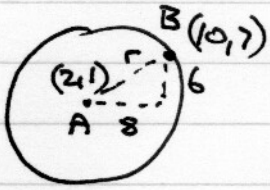
(e) Population each year $= ar^{n-1}$

Amount given " " $= £1 \times ar^{n-1}$, $a = 25000$, $r = 1.03$

$$S_{10} = \frac{a(1-r^n)}{1-r} = 25000\,\frac{(1-1.03^{10})}{1-1.03} = £286596.98$$

$$= £287000 \text{ to nearest } £1000.$$

10 (a)



$(x-a)^2 + (y-b)^2 = r^2$

$r^2 = 8^2 + 6^2 = 100, \quad r = 10$

Circle is $(x-2)^2 + (y-1)^2 = 100$

(b) Gradient of AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-1}{10-2} = \dfrac{6}{8} = 3/4$

$\therefore$ Gradient of tangent $= \dfrac{-1}{(3/4)} = -4/3$

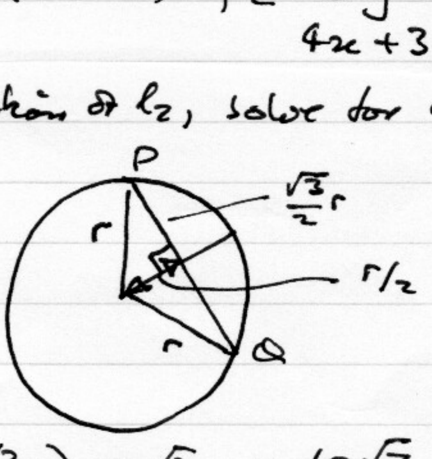
$y - y_1 = m(x - x_1)$

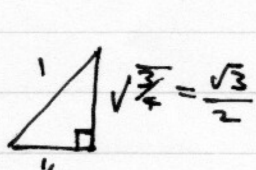
$y - 7 = -4/3(x-10)$, { $3y - 21 = -4x + 40$

$4x + 3y - 61 = 0$ } not essential.

(c) Could find equation of l_2, solve for intersection coordinates etc.

<u>Easier way</u>:





$\therefore AB = 2\left(\frac{\sqrt{3}}{2}r\right) = \sqrt{3}r = 10\sqrt{3}$

<u>nb</u> Only 3 marks! Avoid solving lots of equations!