June 2009 C2

1. "Calculus" means differentiation and integration (C1 notes). i.e. not trapezium rule.

$$\int_a^b x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$\therefore \int_1^4 2x + 3\sqrt{x}\,dx$$

$$= \int_1^4 2x + 3x^{1/2}\,dx = \left[2\left(\frac{x^2}{2}\right) + 3\left(\frac{x^{3/2}}{(3/2)}\right)\right]_1^4$$

$$= \left[x^2 + 2x^{3/2}\right]_1^4$$

$$= \left[4^2 + 2(\sqrt{4})^3\right] - \left[1^2 + 2(\sqrt{1})^3\right] = (16+8) - (1+2)$$

$$= 32 - 3 = 29.$$

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2 (a) Binomial series (cf. book):

$$(a+b)^n = a^n + {}^nC_1\,a^{n-1}b^1 + {}^nC_2\,a^{n-2}b^2 + \ldots$$

n.b. The $\binom{n}{2}$ in the book is "n choose 2" = same as ${}^nC_2$ <u>NOT</u> $\frac{n}{2}$ etc.

For $(2+kx)^7$, $a=2$, $b=kx$, $n=7$ so

$$(2+kx)^7 = 2^7 + {}^7C_1\,2^6(kx)^1 + {}^7C_2\,2^5(kx)^2 + \ldots$$

Note brackets

$$= 128 + 7\times 64(kx) + 21\times 32k^2x^2 + \ldots$$

$$= 128 + 448kx + 672k^2x^2 + \ldots$$

(b) "coefficient of x^2 is 6 times coefficient of x" means $672k^2 = 6(448k)$

$$672k^2 - 2688k = 672(k^2 - 4k) = 0$$

$$\therefore k(k-4) = 0,\quad k = 0 \text{ or } 4.$$

$k=0$ not wanted $\therefore$ $\underline{k = 4}$

3. $f(x) = (3x-2)(x-k) - 8$

(a) $f(k) = (3k-2)(k-k) - 8$

$= 0(3k-2) - 8 = -8$

(b) Let $x - 2 = x - a \therefore a = 2$,

$f(a) = f(2) = (3\times 2-2)(2-k) - 8$

$= 4(2-k) - 8 = 4$ (remainder).

$\div 4$ $\quad 2 - k - 2 = 1$

$-k = 1, \quad k = -1.$

(c) with $k = -1$, $f(x) = (3x-2)(x+1) - 8$

↑Note!

$\therefore f(x) = 3x^2 + 3x - 2x - 2 - 8$

$= 3x^2 + x - 10$

$= 3x^2 + 6x - 5x - 10$

$= 3x(x+2) - 5(x+2)$

$= \underline{(3x-5)(x+2)}$

$\boxed{ac = -30 = 6\times -5}$

4(a)

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{2^x+1}$	1.414	1.554	1.732	1.957	2.236	2.580	3

(calculator 1.5 =
$\sqrt{(2^\wedge \text{ANS}+1)} =$, check answer, then repeat with 2 = and 2.5 = .) .

(b) $\int_a^b y\,dx \approx \frac{h}{2}\left(y_0 + y_n + 2(y_1 + y_2 + \cdots y_{n-1})\right)$

Note brackets!

$h = 0.5$ (difference between x-values in table)

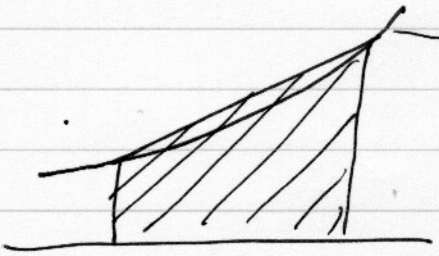
or $h = \frac{b-a}{n} = \frac{3-0}{6}$

↑ 7 coordinates hence 6 strips

(b) $\therefore$ area $\approx \frac{1}{4}\left(1.414+3+2\left(1.554+1.732+1.957+2.236 + 2.580\right)\right)$

$= 6.133$

(c)



curve is concave upwards so tops of trapezia lie above it.

$\therefore$ overestimate.

5(a) $ar^2 = 324$, $ar^5 = 96$ (since n^{th} term $= ar^{n-1}$).

$\therefore \frac{ar^5}{ar^2} = r^3 = \frac{96}{324} = \frac{48}{162} = \frac{24}{81} = \frac{8}{27}$

$\therefore r = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$.

(b) $ar^2 = 324$ so $a = \frac{324}{(2/3)^2} = 324 \times (3/2)^2$

$= \frac{324 \times 9}{4} = 81 \times 9 = 729$.

(c) $S_n = \frac{a(1-r^n)}{1-r}$, $S_{15} = \frac{729(1-(2/3)^{15})}{1-2/3} = 2182$ (to 4 sig fig).

(d) $S_\infty = \frac{a}{1-r} = \frac{729}{(1/3)} = 2187$.

6.

$x^2 - 6x + y^2 + 4y = 12$. Complete the square:

$(x-3)^2 - 3^2 + (y+2)^2 - 2^2 = 12$

$\therefore (x-3)^2 + (y+2)^2 = 25 = 5^2$.

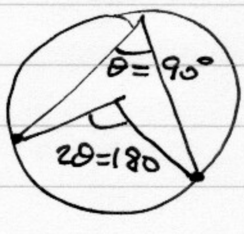
Centre $(3,-2)$ radius 5.

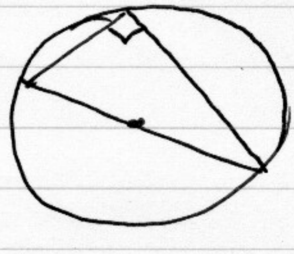
(b) <u>Either</u>: show that length $PQ = 10$ using Pythagoras
$\therefore$ must be a diameter since any other chord would be $< 2r$ in length

<u>or</u> mid-point of $PQ = \left(\frac{-1+7}{2}, \frac{1-5}{2}\right) = (3, -2)$

$=$ centre of circle.
If the chord PQ passes through the centre, it must be a diameter.

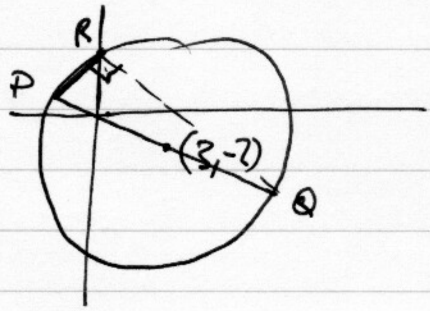
(c) Circle theorems:





A triangle with a diameter as one side must be a right-angled triangle

$\therefore$ The <u>right-angled triangle PRQ must have point R on the circle C</u>.



At $x = 0$,
$0 + y^2 - 0 + 4y = 12$
$y^2 + 4y - 12 = 0$
$(y+6)(y-2) = 0$
$y = 2$ or -6
but $y > 0 \therefore y = 2$

R is at $(0, 2)$.

7. (i) $(1+\tan\theta)(5\sin\theta-2) = 0$.

Note! Like a factorised quadratic.

$\therefore (1+\tan\theta)=0$ or $(5\sin\theta-2)=0$

If $\tan\theta = -1$

$\theta_1 = \tan^{-1}(-1) = 135^\circ$

$\theta_2 = 135-180 = -45^\circ$

or

$AA = \tan^{-1}(1) = 45$

✓S	A
T	C✓

$\theta = 180-45 = \underline{135^\circ}$,

or $\underline{-45^\circ}$.

If $\sin\theta = 2/5$,

$\theta_1 = \sin^{-1}(2/5) = \underline{23.58^\circ}$

$\theta_2 = 180-23.58 = \underline{156.42^\circ}$

or

✓S	A✓
T	C

same.

(ii) $4\sin x = 3\tan x$

$\boxed{\tan x = \frac{\sin x}{\cos x}}$

$4\sin x - 3\frac{\sin x}{\cos x} = 0$

$\times\cos x$:

$\sin x(4\cos x - 3) = 0$

$\therefore \sin x = 0$

$\underline{x = 0^\circ, 180^\circ}$

or $\cos x = 3/4$

S	A✓
T	C✓

$x = \cos^{-1}(3/4)$

$= 41.4^\circ, -41.4^\circ + n360$

$= 41.4^\circ, 318.6^\circ$

8 (a) $\boxed{\text{If } \log_a b = c,\quad a^{\log_a b} = b = a^c}$
(exponential is inverse of the logarithm).

$\therefore \log_2 y = -3, \quad 2^{\log_2 y} = y = 2^{-3} = \frac{1}{8}$.

(b) $\log_2 32 = \log_2(2^5) = 5.$ $\log_2 16 = \log_2(2^4) = 4.$

$\dfrac{4+5}{\log_2 x} = \log_2 x$ $\therefore (\log_2 x)^2 = 9$

$\log_2 x = \sqrt{9} = \pm 3$

$\therefore \log_2 x = -3$, $\underline{x = 1/8}$ as (a)

or

$\log_2 x = 3$, $\underline{x = 2^3 = 8}$.

Note: question asks for "VALUES" (not just one)!

9(a) S = 2 sectors + 2 flat sides + 1 curved

$= 2(\tfrac{1}{2}r^2\theta) + 2hr + h(r\theta)$, $\theta = 1$

$= r^2 + 3hr$.

Constraint $V = h(\tfrac{1}{2}r^2\theta) = \tfrac{1}{2}r^2h = 300$ $\therefore h = \dfrac{600}{r^2}$

$\therefore S = r^2 + 3r\left(\dfrac{600}{r^2}\right) = r^2 + \dfrac{1800}{r} = r^2 + 1800r^{-1}$

(b) $dS/dr = 2r - 1800r^{-2}$.

where $dS/dr = 0$, $2r = \dfrac{1800}{r^2}$ $\therefore r = \dfrac{900}{r^2}$,

$r^3 = 900$, $r = \sqrt[3]{900} = 9.655$ cm.

(c) $d^2S/dr^2 = 2 + 3600r^{-3}$. For any positive r, this is positive.

$\therefore$ S is a minimum.

(d) At $r = 9.655$, $S = r^2 + \dfrac{1800}{r} = 279.65\ \text{cm}^2$

$= 280\ \text{cm}^2$ to nearest cm^2