C2 JUNE 2005

1. $y = 2x^2 - 12x$, stationary point when $dy/dx = 0$

$dy/dx = 4x - 12$ $\therefore$ $4x = 12$, $x = 3$, $y = 2(3^2) - 12\times 3$
$= 18 - 36 = -18$

$\therefore (3, -18)$.

2. (a) Solve $5^x = 8$. Either (i) $x = \log_5(8) = 1.29$ (3s.f.)

or (ii) $\log_{10}(5^x) = \log_{10}(8)$

$x\log_{10}(5) = \log_{10}(8)$

$x = \frac{\log 8}{\log 5} = 1.292$

$= 1.29$ to 3 sig.figs.

(b) $\log_2(x+1) - \log_2(x) = \log_2(7)$.

Use $\log A - \log B = \log(A/B)$

$\therefore \log_2\left(\frac{x+1}{x}\right) = \log_2(7) \rightarrow \frac{x+1}{x} = 7$,

$x + 1 = 7x$, $1 = 6x$, $x = 1/6$

3(a) Let $f(x) = 2x^3 + x^2 - 25x + 12$

"If $(x-a)$ is a factor, $f(a) = 0$", let $x + 4 = x - a$

so $a = -4$.

$f(a) = f(-4) = 2(-4)^3 + (-4)^2 - 25\times -4 + 12$

$= -128 + 16 + 100 + 12 = -128 + 128 = 0$ ✓

(b) Use long division to convert $f(x)$ into $(x+4)$(quadratic):

$$
\begin{array}{r|l}
 & 2x^2 - 7x + 3 \\
x + 4 & 2x^3 + x^2 - 25x + 12 \\
 & 2x^3 + 8x^2 \quad - \\
 & \quad -7x^2 - 25x + 12 \\
 & \quad -7x^2 - 28x \quad - \\
 & \qquad 3x + 12 \\
 & \qquad 3x + 12 \quad - \\
 & \qquad\quad 0
\end{array}
$$

Factorising $2x^2 - 7x + 3$:

$ac = 6 = -1\times -6$

$2x^2 - x - 6x + 3$

$= x(2x-1) - 3(2x-1)$

$= (x-3)(2x-1)$

or $\left(2x - \frac{1}{1}\right)\left(1x - \frac{6}{2}\right)$

$= (2x-1)(x-3)$.

$\therefore 2x^3 + x^2 - 25x + 12 = (x+4)(x-3)(2x-1)$.

4.(a) $(1+px)^{12} = 1 + {}^{12}C_1(px) + {}^{12}C_2(px)^2 + \dots$

$= 1 + 12px + 66p^2x^2 + \dots$

(b) Given $(1+px)^{12} = 1 - qx + 11qx^2$,

$12p = -q \rightarrow q = -12p$

$66p^2 = 11q$

$= 11(-12p)$

$\therefore 6p^2 = -12p, \quad p^2 = -2p, \quad p^2 + 2p = 0$

$p(p+2) = 0, \quad p = 0 \text{ or } -2.$

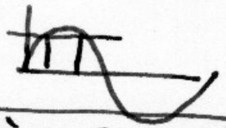
p non-zero $\therefore \underline{p = -2}, \quad q = -12 \times -2 \; \underline{= 24}.$

[or $\dfrac{66p^2}{12p} = \dfrac{11}{2}p = \dfrac{11q}{-q} = -11$, $p = -2$.]

5.(a) $\sin(x+10) = \sqrt{3}/2, \quad 0 \le x \le 180$

$\therefore x + 10 = \sin^{-1}(\sqrt{3}/2) = 60°$ & repeats $180-60, \pm n360$

$= 60°, 120°.$



$x = 60 - 10 = 50°,$

$120 - 10 = 110°.$

$\boxed{\text{sine } \theta_2 = 180 - \theta_1}$

(b) $\cos 2x = -0.9 \quad \therefore 2x = \cos^{-1}(-0.9) = 154.16°$ & repeats.

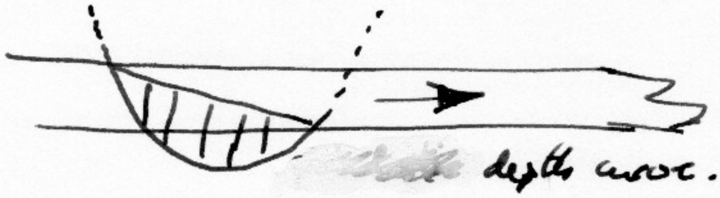
$\boxed{\text{cosine } \theta_2 = -\theta_1}$

$2x = \pm 154.16 + n360$

halve: $x = \pm 77.08 + n180$

$= \underline{77.1°}$ (1 d.p., $0 \le x \le 180$).

6.



depth curve.

<u>Use ANS button</u>

0 =

ANS × √(20 − ANS) ÷ 10 =

(a)

x	0	4	8	12	16	20
y	0	1.6	2.771	3.394	3.2	0

so $h = 4$ (= x interval).

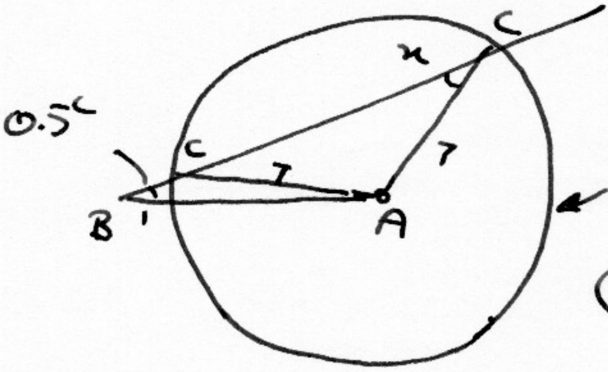
(b) $A \approx \frac{h}{2}\left((y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)\right) = 43.86\,m^2$

(c) 2m/s speed, so water moves along $2 \times 60 = 120$m in 1 minute

$\therefore$ Volume flow rate $= 43.86 \times 120 = 5263.2\ m^3/min$

$= 5260\ m^3/min$ (3 s.figs).

7. <u>Radians mode</u>



construction circle radius 7

(a) sine rule, $\frac{\sin x}{8} = \frac{\sin 0.5}{7}$

$\sin x = \frac{\sin 0.5}{7} \times 8 = 0.5479$

$= \underline{0.548}$ (3 d.p.)

(b) $x = \sin^{-1}(0.5479)$

$= 0.57987, \ \pi - 0.57987$

$= 0.58, \ 2.56^c$ (2 d.p.)

8(a) Circle $x^2 + y^2 - 10x + 9 = 0$

$= (x-5)^2 - 25 + y^2 + 9$

$\therefore (x-5)^2 + y^2 = 25 - 9 = 16$

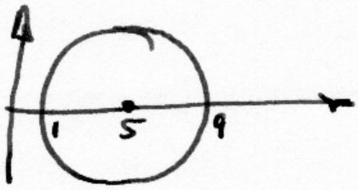
Centre $(5, 0)$

(b) radius $\sqrt{16} = 4$

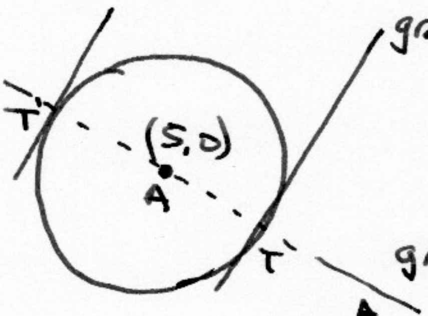
(c) At $y = 0$, $(x-5)^2 = 16$, $x - 5 = \pm 4$

$x = -4 + 5 = 1$ or $+4 + 5 = 9$

$\therefore$ Cuts at $(1, 0)$ and $(9, 0)$



(d)



gradient $7/2$

gradient of normal $\frac{-2}{7}$

line $y - 0 = -2/7\,(x - 5)$

$y = -2/7\,x + \frac{10}{7}$

9.(a) $S_n = a + ar + ar^2 + \dots ar^{n-1}$

$rS_n = \quad ar + ar^2 + \dots ar^{n-1} + ar^n$

$\therefore S_n - rS_n = a - ar^n$

$(1-r)S_n = a(1-r^n), \qquad S_n = \dfrac{a(1-r^n)}{1-r}$

(b) Let n = year $- 2004$, $n=1$ in 2005 etc

In 2008 $n=4$, $ar^{n-1} = ar^3 = 35000(1.04)^3$

$= 39370.24$

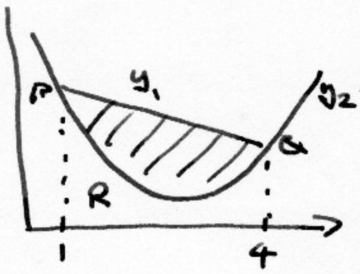
$= £39400$ to nearest £100.

(c) In 2024, $n = 2024 - 2004 = 20$

$S_{20} = \dfrac{35000(1-1.04^{20})}{1-1.04} = 1,042,232.75$

$= £1042000$ to nearest £1k.

10.(a)



$y_2 = 2x + \dfrac{8}{x^2} - 5$

At $x=1$, $y_2 = 2+8-5 = 5$

At $x=4$, $y_2 = 8 + \frac{1}{2} - 5 = 3\frac{1}{2}$

Area R under curve $= \int_1^4 2x + 8x^{-2} - 5\,dx = \left[x^2 - 8x^{-1} - 5x\right]_1^4$

$= (16-2-20)-(1-8-5) = -6-(-12) = 6$

Area of trapezium $= \frac{1}{2}(5+3\frac{1}{2})(4-1) = \dfrac{8\frac{1}{2}\times 3}{2} = \dfrac{51}{4} = 12.75$

$\therefore$ shaded area $= 12.75 - 6 = 6.75$.

(b) $y = 2x + 8x^{-2} - 5, \quad dy/dx = 2 - 16x^{-3}$

Increasing where $dy/dx > 0 \;\therefore\; 2 > 16x^{-3}$;

$\frac{1}{8} > x^{-3}, \quad \frac{x^3}{8} > 1, \quad x^3 > 8, \quad \underline{x > 2}$.