C2 JAN 2011

1. $f(x) = x^4 + x^3 + 2x^2 + ax + b.$

a) "Remainder theorem, when $\div$ by $(x-a)$ remainder is $f(a)$"

$\therefore f(1) = 1^4 + 1^3 + 2\times 1 + a + b = 7$

$a + b = 7 - 4 = 3$

b) $f(-2) = (-2)^4 + (-2)^3 + 2(-2)^2 - 2a + b$

$= 16 - 8 + 8 - 2a + b = -8$

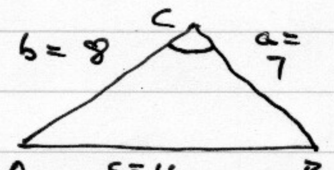
$\therefore \left.\begin{array}{l} -2a + b = -8 - 16 = -24 \\ 2a + 2b = 6 \end{array}\right\}$ add

$a + b = 3 \quad \therefore$

$3b = -18, \quad b = -6,$

$a = 3 + 6 = 9$

2. a)



Need 3 lengths & 1 angle $\Rightarrow$ use cosine rule.

$a^2 = b^2 + c^2 - 2bc\cos A$

Swop a, b, c: $\quad \therefore c^2 = a^2 + b^2 - 2ab\cos C$

$11^2 = 7^2 + 8^2 - 2\times 7\times 8\cos C = 49 + 64 - 112\cos C$

$\therefore \cos C = \dfrac{49 + 64 - 121}{112} = -\dfrac{1}{14}, \quad C = 1.6423^c$

$= 1.64$ to 3 s.f.

b) Area $= \frac{1}{2}ab\sin C = \frac{1}{2}\times 7\times 8\sin C$

$= 27.928\text{ cm}^2 = 27.9\text{ cm}^2$ to 3 s.f.

3 (a) n^{th} term is ar^{n-1} $\quad \therefore 2^{nd}$ is $ar = 750$

5^{th} is $ar^4 = -6$

$\dfrac{ar^4}{ar} = r^3 = \dfrac{-6}{750} = \dfrac{-1}{125} \quad \therefore r = \sqrt[3]{\dfrac{-1}{125}} = -\frac{1}{5}$

(b) $a = \dfrac{750}{r} = 750 \times \left(\dfrac{-5}{1}\right) = -3750$

(c) $|r| < 1$ so S_∞ exists. $\quad S_\infty = \dfrac{a}{1-r} = \dfrac{-3750}{1.2} = -3125$

4. (a) $y = (x+1)(x-5)$ so $y = 0$ at $x = -1$ (A), $x = 5$ (B)

(b) $\displaystyle\int_{-1}^{5} y\,dx = \int_{-1}^{5} x^2 - 4x - 5\,dx = \left[\frac{x^3}{3} - 2x^2 - 5x\right]_{-1}^{5}$

$= \left(\frac{125}{3} - 50 - 25\right) - \left(-\frac{1}{3} - 2 + 5\right) = -36 \quad \therefore$ area $= 36$

5(a) By definition (formula book), $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

$\therefore \binom{40}{4} = \frac{40!}{4!(40-4)!}$, $b = 36$

(b) $(1+x)^{40} = 1 + {}^{40}C_1 x^1 + \dots {}^{40}C_4 x^4 + {}^{40}C_5 x^5 + \dots$

$= 1 + 40x + \quad px^4 + \quad qx^5 + \dots$

$\therefore \frac{q}{p} = \frac{40C5}{40C_4} = \frac{36}{5}$ (calculator).

or $\frac{\left(\frac{40!}{5!\ 35!}\right)}{\left(\frac{40!}{4!\ 36!}\right)} = \frac{4!\ 36!}{5!\ 35!} = \frac{36}{5}$

6(a) 2 =

$5 \div (3 \times \text{ANS}\,\boxed{x^2} - 2) =$, repeat for 2.5, 2.75

x	2	2.25	2.5	2.75	3
y	0.5	0.38	0.3	0.24	0.2

(b) $h = 0.25$ (x interval in table), $\underline{2.25 - 2 = 0.25 = h}$

$\int_2^3 \frac{5}{3x^2-2}\,dx \approx \frac{h}{2}\left(y_0 + y_4 + 2(y_1 + y_2 + y_3)\right)$

$= 0.3175 \qquad = 0.32$ to 2 sig. figs.

(c) Shaded area $= 0.3175 -$ area of triangle

$= 0.3175 - \frac{1}{2} \times 1 \times 0.2$

$= 0.2175$, 0.22 to 2 sig. figs.

7.(a) $\cos^2 x = 1 - \sin^2 x$

$\therefore 3\sin^2 x + 7\sin x = \cos^2 x - 4 = 1 - \sin^2 x - 4 = -\sin^2 x - 3$

$4\sin^2 x + 7\sin x + 3 = 0$

(b) $ac = 12 = 4 \times 3 \qquad \left(4\sin x + \frac{3}{1}\right)\left(\sin x + \frac{4}{4}\right) = 0$

$\sin x = -3/4$ or -1

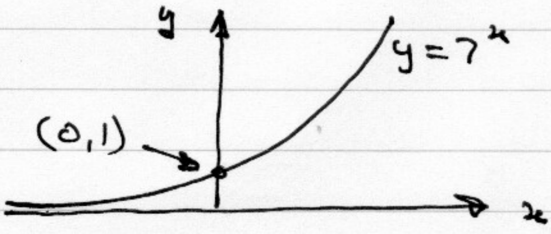
$\sin x = -3/4, \quad x = -48.6^\circ,\ 180-(-48.6^\circ) + n360$

$= -48.6,\ 228.6 + n360$

$= 311.4^\circ,\ 228.6^\circ$ in $0 \to 360$ range.

$\sin x = -1, \quad x = 270^\circ.$

8(a)



(b) $7^{2x} = (7^x)^2$, let $y = 7^x$ then

$7^{2x} - 4(7^x) + 3 = y^2 - 4y + 3 = 0$

$= (y-3)(y-1)$

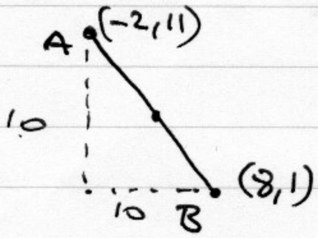
$\therefore\ 7^x = 1 \Rightarrow x = 0,$

$7^x = 3, \quad x \log 7 = \log 3, \quad x = \dfrac{\log 3}{\log 7} = 0.5646$

$= 0.56$ to 2dp

(or $x = \log_7(3) = 0.56$).

9(a)



Midpoint $\left(\dfrac{-2+8}{2}, \dfrac{11+1}{2}\right)$

$= (3, 6)$.

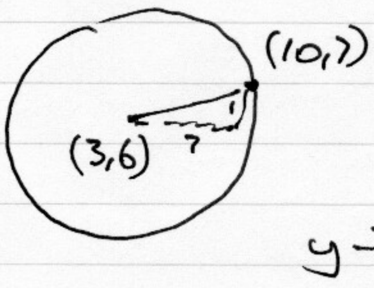
(b) Diameter $= \sqrt{10^2+10^2} = \sqrt{200} = 10\sqrt{2}$, radius $= 5\sqrt{2}$

$(x-3)^2 + (y-6)^2 = (5\sqrt{2})^2 = 50$

(c) At $(10,7)$, $(10-3)^2 + (7-6)^2 = 7^2 + 1^2 = 49 + 1 = 50$ ✓

$\therefore$ equation satisfied.

(d)



gradient of radius $= \dfrac{7-6}{10-3} = \dfrac{1}{7}$

$\therefore$ gradient of tangent $= -7$

$y - 7 = -7(x-10) = -7x + 70$

$\therefore\ y = -7x + 77$

10.(a) $V = 4x(5-x)^2 = 4x(25-10x+x^2)$

$$= 100x - 40x^2 + 4x^3$$

$$\therefore \frac{dV}{dx} = 100 - 80x + 12x^2$$

(b) At max volume, $dV/dx = 0$

$$\therefore 12x^2 - 80x + 100 = 0$$

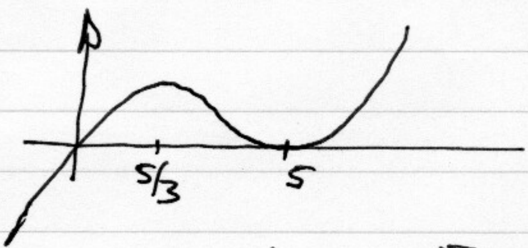
$$3x^2 - 20x + 25 = 0$$

$$ac = 75 = -5 \times -15$$

$$\left(3x - \frac{5}{1}\right)\left(1x - \frac{15}{3}\right) = (3x-5)(x-5) = 0$$

$$x = 5/3 \text{ or } 5$$

V is a cubic:



$\therefore$ the maximum is at $x = 5/3$,

then $V = \frac{20}{3}\left(\frac{10}{3}\right)^2 = \frac{2000}{27} = 74.07 \text{ cm}^3$

(c) $d^2V/dx^2 = -80 + 24x$.

At $x = 5/3$, $d^2V/dx^2 = -40$ $\therefore$ maximum

At $x = 5$, $d^2V/dx^2 = 40$ $\therefore$ minimum.

nb Do not try to solve $d^2V/dx^2 = 0$

$\frac{dV}{dx} = 0$ defines a point with zero gradient 

$d^2V/dx^2 = 0$ defines a point with zero curvature.

This is not what you want.