C2 JAN 2010

1. $(a+b)^n = a^n + {}^nC_1 a^{n-1}b^1 + {}^nC_2 a^{n-2}b^2 + \ldots$

$\therefore (3-x)^6 = 3^6 + {}^6C_1(3^5)(-x) + {}^6C_2(3^4)(-x)^2 + \ldots$

$= 729 + 6\times 243(-x) + 15(81)x^2 + \ldots$

$= 729 - 1458x + 1215x^2 + \ldots$

2. (a) $\cos^2 x + \sin^2 x \equiv 1$ $\therefore \cos^2 x = 1 - \sin^2 x$

(Pythagoras).

$5\sin x = 1 + 2\cos^2 x = 1 + 2(1 - \sin^2 x)$

$= 1 + 2 - 2\sin^2 x$

$= 3 - 2\sin^2 x$

$\therefore 2\sin^2 x + 5\sin x - 3 = 0$

(b) Let $y = \sin x$, $2y^2 + 5y - 3 = 0$ $ac = -6 = 6\times -1$

$(2y - \frac{1}{1})(1y + \frac{6}{2}) = (2\sin x - 1)(\sin x + 3) = 0$

$\therefore$ $\sin x = -3$ (no real roots),

$\sin x = 1/2$

$x = \sin^{-1}(1/2) = 30°, 180-30 + n360$

$= 30°, 150°$ in range $0 \to 360$.

3(a) $f(x) = 2x^3 + ax^2 + bx - 6$

Remainder when $\div (2x-1)$ same as remainder when $\div (x - 1/2)$

Let $(x-a) = (x - 1/2)$, $a = 1/2$.

$f(a) = f(1/2) = 2(1/2)^3 + a(1/2)^2 + b(1/2) - 6$

$= \frac{1}{4} + \frac{1}{4}a + 1/2 b - 6 = -5$ (remainder).

($\times 4$) $1 + a + 2b - 24 = -20$

$a + 2b = 3$

Remainder when $\div$ by $(x+2)$ is 0 $\therefore f(-2) = 0$

$f(-2) = 2(-2)^3 + a(-2)^2 + b(-2) - 6$

$= -16 + 4a - 2b - 6 = -22 + 4a - 2b = 0$

$4a - 2b = 22$

Adding: $(a+2b) + (4a-2b) = 3 + 22$

$5a = 25$, $a = 5$

$4a-2b=22,\quad 2a-b=11,\quad b=2a-11=-1$

3(b) $f(x)=2x^3+5x^2-x-6$

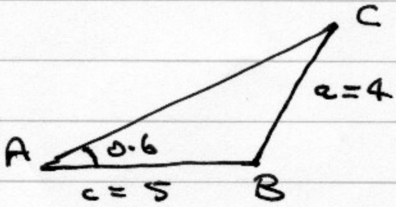
$$\begin{array}{r l}
 & 2x^2+x-3 \\
x+2\ \big) & 2x^3+5x^2-x-6 \\
 & \underline{2x^3+4x^2} \quad - \\
 & x^2-x-6 \\
 & \underline{x^2+2x} \quad - \\
 & -3x-6 \\
 & \underline{-3x-6} \quad - \\
 & 0
\end{array}$$

$ac=-6=3\times -2$

$\therefore\ 2x^2+x-3=\left(2x+\frac{3}{1}\right)\left(1x-\frac{2}{2}\right).$

$f(x)=(x+2)(2x+3)(x-1).$

4.(a)



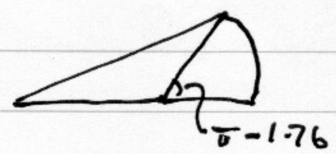
Think: cosine rule could find side b (solving quadratic) but not helpful.
$\therefore$ use sine rule, find angle C, then B.

$\frac{\sin C}{c}=\frac{\sin A}{a},\quad \sin C=\left(\frac{\sin 0.6}{4}\right)\times 5=0.706,$

$C=\sin^{-1}(\text{ANS})=0.784$ radians

$\angle ABC=\pi-0.6-\text{ANS}=1.758,\ =1.76$ radians to 3 s.f.

(b)



Total area = triangle area + sector area
"$\frac{1}{2}ab\sin\theta$" "$\left(\frac{1}{2}r^2\theta\right)$"

$=\frac{1}{2}\times 5\times 4\sin 1.76+\frac{1}{2}\times 4^2(\pi-1.76)$

$=20.894\text{ cm}^2$

$=20.9\text{ cm}^2$ to 3 s.f.

5(a) $\log_x(64)=2,\quad x^{\log_x 64}=x^2\ \therefore\ 64=x^2;\quad x=\sqrt{64}=8$

(b) $\log_2(11-6x)=2\log_2(x-1)+3$
$=\log_2(x-1)^2+3$

Need "log = number" form:

$$\log_2(11-6x) - \log_2(x-1)^2 = 3$$

$$= \log_2\left[\frac{11-6x}{(x-1)^2}\right]$$

$$\therefore \quad \frac{11-6x}{(x-1)^2} = 2^3 = 8$$

$$11-6x = 8(x-1)^2 = 8(x^2-2x+1) = 8x^2-16x+8.$$

$$\therefore \quad 8x^2-16x+6x+8-11 = 0$$

$$8x^2-10x-3 = 0 \qquad ac = -24 = -12\times 2$$

$$\left(4x+\tfrac{2}{2}\right)\left(2x-\tfrac{12}{4}\right) = (4x+1)(2x-3) = 0$$

$$\therefore \; x = -\tfrac{1}{4},\; \tfrac{3}{2}$$

6(a) After 3 years, value $= 18000 \times 0.8^3 = £9216$

(b) After n years, value $= 18000 \times 0.8^n$

$$18000 \times 0.8^n < 1000$$

$$0.8^n < \frac{1}{18}$$

$$n \log_{10}(0.8) < \log_{10}(18^{-1})$$

$$'' \quad < -\log_{10} 18$$

$\log_{10}(0.8)$ is negative, so $n > \dfrac{-\log_{10}(18)}{\log_{10}(0.8)}$, $n > 12.95$

$\therefore n = 13$.

(c) Geometric series, $a = 200$, $r = 1.12$

5th term $ar^{n-1} = ar^4 = 200 \times (1.12)^4 = £314.70$

(d) $S_{15} = \dfrac{a(1-r^n)}{1-r} = \dfrac{200(1-1.12^{15})}{1-1.12} = £7455.94$

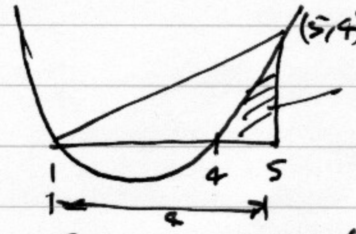
7(a)  $y = x^2-5x+4$
$= (x-4)(x-1)$ $\quad \therefore$ At $y=0$, $x = 1$ or 4

L is $(1,0)$, M is $(4,0)$.

(b) At $x=5$, $y = (5-4)(5-1) = 1\times 4 = 4$

$\therefore (5,4)$ is on C.

(c) $\displaystyle\int x^2-5x+4\,dx = \frac{x^3}{3} - \frac{5x^2}{2} + 4x + c$

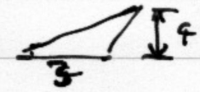
7 (d) 

area $= \int_4^5 x^2-5x+4\,dx = \left[\frac{x^3}{3}-\frac{5}{2}x^2+4x\right]_4^5$

$= \left(\frac{125}{3}-\frac{125}{2}+20\right)-\left(\frac{64}{3}-40+16\right) = \frac{11}{6}$

∴ R = area of triangle − $11/6$

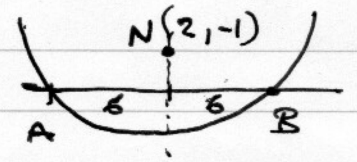
$= \frac12 \times 4 \times 4 - 11/6 = 37/6 = 6\frac16$

<u>check</u>: just slightly > a triangle 

, area 6. ✓

8. (a) $(x-a)^2+(y-b)^2=r^2$ so $(x-2)^2+(y+1)^2=(13/2)^2$ has centre $(2,-1)$.

(b) $r=\sqrt{\frac{169}{4}}=\frac{13}{2}=6\frac12$

(c) 

At A, $x=2-6=-4$

$(-4-2)^2+(y+1)^2=(13/2)^2$

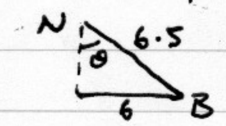
$(y+1)^2=6.5^2-6^2=\frac{25}{4}$

$y+1=\pm 5/2$,

$y=-1\pm 5/2=-3\frac12,\ 1\frac12$.

∴ A is $(-4, -3\frac12)$

B is $(8, -3\frac12)$

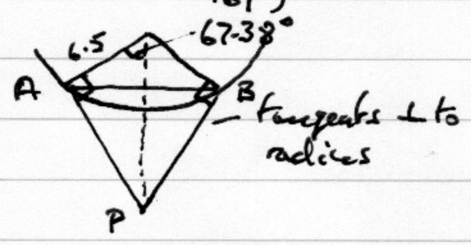
(d) 

$\theta=\sin^{-1}(6/6.5)=67.38°$,

$\angle ANB = 2\theta = 134.8°$ to 1d.p.

<u>or</u> cosine rule, $n^2=a^2+b^2-2ab\cos N$, $12^2=6.5^2+6.5^2-2\times6.5^2\cos N$,

$\cos N=-\frac{119}{169}$, $N=\angle ANB=134.8°$.

(e) 

$\tan 67.38° = \frac{AP}{6.5}$, $AP = 6.5\tan 67.38$

$=15.5999$

$=15.6$ units to 3s. figs

9. (a) $y=12x^{1/2}-x^{3/2}-10$, $dy/dx = 12(\frac12 x^{-1/2})-\frac32 x^{1/2} = 6x^{-1/2}-\frac32 x^{1/2}$

At turning point, $dy/dx=0$ ∴ $6x^{-1/2}=\frac32 x^{1/2}$, $6=\frac32 x$, $x=4$

$y=12\sqrt4-4\sqrt4-10=24-8-10=6$, t.point is $(4,6)$.

(b) $\frac{d^2y}{dx^2}=6\left(-\frac12 x^{-3/2}\right)-\frac32\left(\frac12 x^{-1/2}\right)=-3x^{-3/2}-\frac34 x^{-1/2}$

(c) At $x=4$, $d^2y/dx^2=-3(4\sqrt4)^{-1}-\frac34\left(\frac{1}{\sqrt4}\right)=-\frac38-\frac38=-\frac34$

$d^2y/dx^2<0$ ∴ T.P. is a maximum