C2 JAN 2008

1(a) $f(x) = x^3 - 2x^2 - 4x + 8$

(i) Dividing by $(x-a) = (x-3) \Rightarrow a = 3$

$f(3) = 3^3 - 2\times 3^2 - 4\times 3 + 8$

$= \text{remainder} = 5$

(ii) Dividing by $(x-a) = (x+2) \Rightarrow a = -2$

Remainder $= f(-2) = (-2)^3 - 2(-2)^2 - 4(-2) + 8 = 0$

(b)
$$
\begin{array}{r l}
 & x^2 - 4x + 4 \\
x+2 \,\big) & x^3 - 2x^2 - 4x + 8 \\
 & x^3 + 2x^2 \qquad\qquad - \\
\hline
 & -4x^2 - 4x + 8 \\
 & -4x^2 - 8x \qquad - \\
\hline
 & 4x + 8 \\
 & 4x + 8 \quad - \\
\hline
 & 0
\end{array}
$$

$x^2 - 4x + 4 = (x-2)^2$

so $x^3 - 2x^2 - 4x - 8 = (x+2)(x-2)^2 = 0$

$x = -2$ or $x = +2$ (repeated root).

2. 4th term $ar^3 = 10$

7th term $ar^6 = 80$

(a) $\dfrac{ar^6}{ar^3} = r^3 = 8 \Rightarrow r = 2$

(b) $a = \dfrac{ar^3}{r^3} = \dfrac{10}{8} = 1.25$

(c) $S_{20} = \dfrac{a(1-r^n)}{1-r} = 1.25\dfrac{(1-2^{20})}{1-2} = 1310719$

to nearest integer.

3. (a) $\left(1+\cancel{\tfrac{4x}{4}}\tfrac{x}{2}\right)^{10} = 1 + 10\left(\frac{x}{2}\right) + \frac{10\times 9}{2}\left(\frac{x}{2}\right)^2 + \frac{10\times 9\times 8}{6}\left(\frac{x}{2}\right)^3 + \ldots$

$+ \cancel{\frac{10\times 9\times 8\times 7}{24}\left(\frac{x}{2}\right)^4}$

$= 1 + 5x + \frac{90}{8}x^2 + \frac{720}{48}x^3 + \cdots$

$= 1 + 5x + 11.25x^2 + 15x^3 + \cdots$

(b) Let $x = 0.01$ so $1+\frac{x}{2} = 1.005$

Then $(1.005)^{10} = 1 + 5\times.01 + 11\tfrac{1}{4}\times 10^{-4} + 15\times 10^{-6}$

$= 1.05114$

4.(a) Using $\sin^2\theta + \cos^2\theta = 1$:

$$3\sin^2\theta - 2\cos^2\theta = 3\sin^2\theta - 2(1-\sin^2\theta)$$

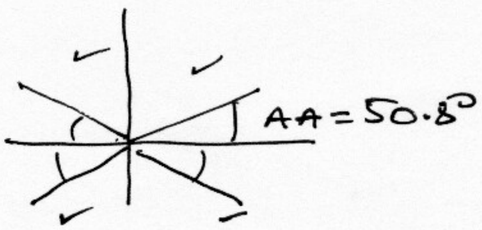
$$= 5\sin^2\theta - 2 = 1$$

$$\therefore 5\sin^2\theta = 3$$

(b) $\sin^2\theta = 3/5$, $\sin\theta = \pm\sqrt{\frac{3}{5}}$,

$\theta = 50.8°$, $180 - 50.8 = 129.2°$,

$-50.8°$, $180 + 50.8 = 230.8°$



5. $a = 3b$

$\log_3 a + \log_3 b = 2$

$\log_3 a = \log_3(3b) = \log_3 b + 1$

$\therefore (\log_3 b + 1) + \log_3 b = 2$

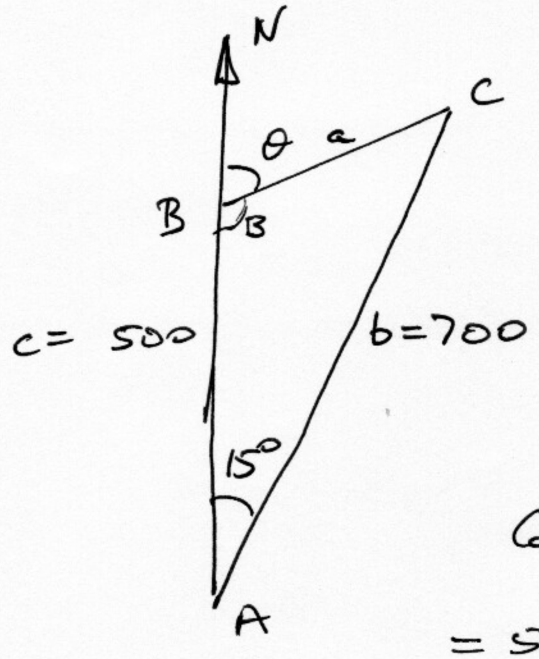
$2\log_3 b = 1$, $\log_3 b = 1/2$,

$b = \sqrt{3}$

$a = 3\sqrt{3}$

6.

(a)



Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$= 500^2 + 700^2 - 2 \times 500 \times 700 \cos 15$

a = distance BC $= 252.7$m

$= 253$m to 3 sig. figures

(b) B is clearly obtuse ($700^2 > 253^2 + 500^2$)

Can use sine rule:

$\sin B = b\left(\frac{\sin A}{a}\right) = 0.717,$

$B = 45.8°$ or $180 - 45.8$

$\times$ ✓

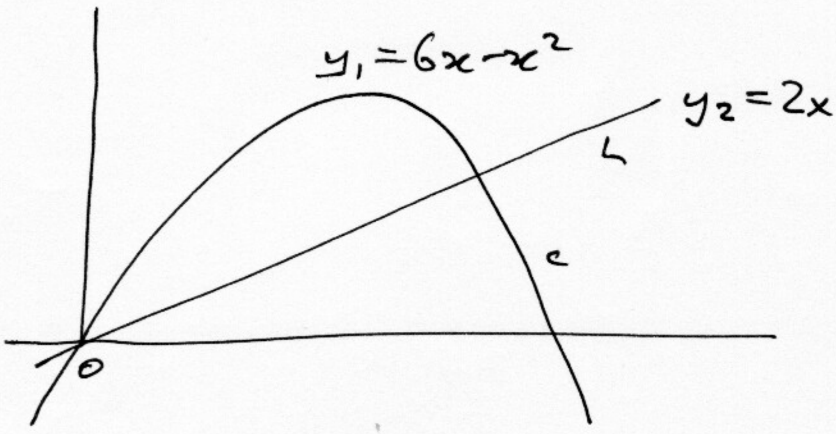
so $\theta = 180 - B = 45.8°$

or cosine rule:

$700^2 = 500^2 + 252.7^2 - 2 \times 500 \times 252.7 \cos B$

$\Rightarrow B = 134.2°, \quad \theta = 180 - B = 45.8°$

7.



(a) where $y_1 = 0$, $\quad 6x - x^2 = x(6-x) = 0$

$\therefore x = 0 \text{ or } 6$

(b) when $y_1 = y_2$

$2x = 6x - x^2$

$x^2 - 4x = x(x-4) = 0$

$x = 0, 4 \qquad$ so $(0,0)$ and $(4, 8)$

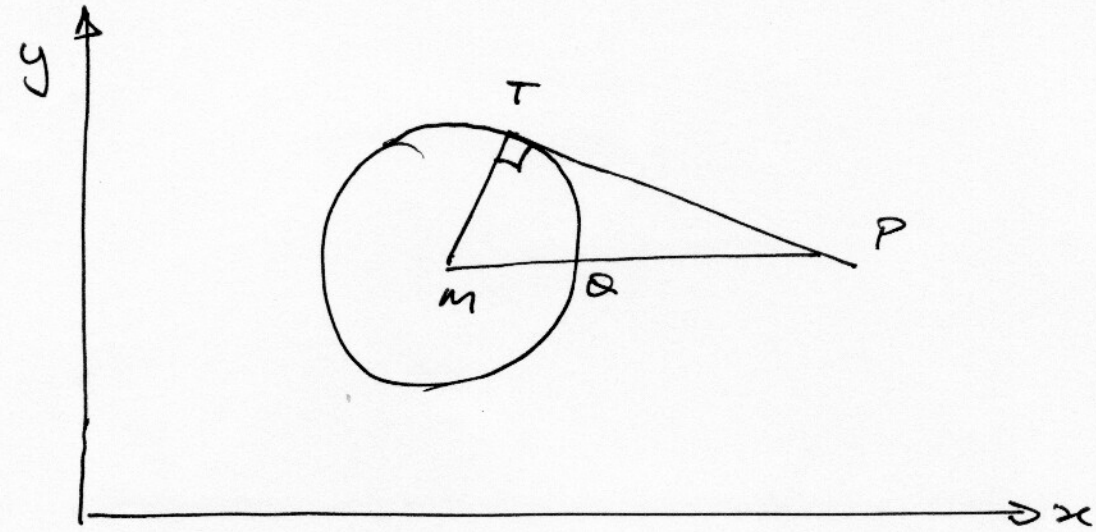
$y = 2x = 0, 8$

(c) $\displaystyle\int_0^4 y_1 - y_2 \, dx = \int_0^4 4x - x^2 \, dx = \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \cancel{8 - \frac{8}{3} = 8\times\frac{2}{3} = \frac{16}{3} = 5\tfrac{1}{3} = \text{area of } R}$.

$= 32 - \frac{64}{3} = 10\tfrac{2}{3} = \text{area of } R$

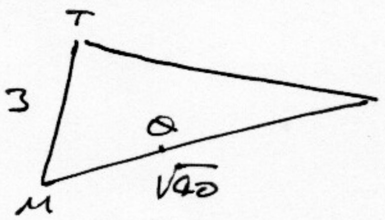
8.



(a) Circle centre $(6,4)$, radius 3:

$$(x-6)^2+(y-4)^2 = 3^2 = 9$$

(b)



Length $MN = \sqrt{(12-6)^2+(6-4)^2}$

$= \sqrt{6^2+2^2} = \sqrt{40}$

$\cos TMQ = \frac{3}{\sqrt{40}}$, $\therefore \angle TMQ = 61.68^\circ = 1.0766$ radians

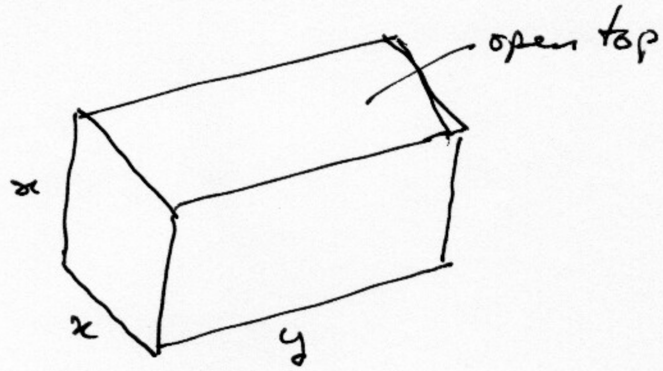
(c) Area of shaded bit = area of triangle TMP

− area of sector TMQ

$= \frac{1}{2} \times 3 \times \sqrt{40} \sin 1.0766 - \frac{1}{2} \times 3^2 \times 1.0766$

$= 3.507$

9.



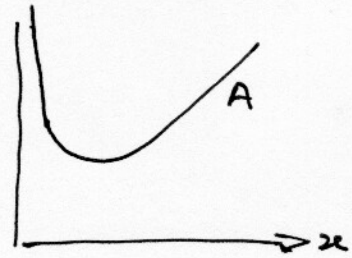
(a) Volume $= 100\,m^3 = x^2 y \quad \therefore \; y = \dfrac{100}{x^2}$

Surface area $= \underbrace{2x^2}_{\text{two ends}} + \underbrace{3xy}_{\text{base + 2 sides}} = A$

Substitute for y: $A = 2x^2 + 3x\left(\dfrac{100}{x^2}\right)$

$$= 2x^2 + \frac{300}{x}$$

(b)



$$\frac{dA}{dx} = 4x - 300x^{-2}$$

When A is stationary, $dA/dx = 0 \Rightarrow 4x = \dfrac{300}{x^2}$

$4x^3 = 300, \quad x = \sqrt[3]{\dfrac{300}{4}} = 4.217\,m$

(c) $\dfrac{d^2A}{dx^2} = \dfrac{d}{dx}(4x - 300x^{-2}) = 4 + 600x^{-3}$

For $x > 0$, $\dfrac{d^2A}{dx^2} > 0$ so the stationary point is a minimum.

(d) $A = 2(4.217)^2 + \dfrac{300}{4.217} = 106.7\,m^2$