JAN 2006 C2

1. $f(x) = 2x^3 + x^2 - 5x + c$

(a) $f(1) = 0 \quad \therefore \quad 2+1-5+c = 0, \quad -2+c=0, \quad c = 2.$

(b) Since $f(1)=0$, $x-1$ is a factor.

$$
\begin{array}{r}
2x^2 + 3x - 2 \\
x-1 \,\big)\, \overline{2x^3 + x^2 - 5x + 2} \\
\underline{2x^3 - 2x^2} \quad - \\
3x^2 - 5x + 2 \\
\underline{3x^2 - 3x} \quad - \\
-2x + 2 \\
\underline{-2x + 2} \quad - \\
0
\end{array}
$$

$2x^2+3x-2 \rightarrow ac = -4$
$\downarrow \qquad = 4 \times -1$

$\left(2x - \frac{1}{1}\right)\left(1x + \frac{4}{2}\right) = (2x-1)(x+2)$

$\therefore f(x) = (x-1)(2x-1)(x+2)$

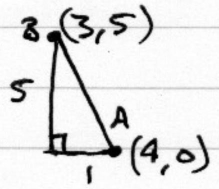
(c) Remainder theorem, remainder when $\div$ by $(2x-3)$ is same as remainder when $\div$ by $(x - 3/2)$,
remainder $= f(3/2) = 2(3/2)^3 + (3/2)^2 - 5(3/2) + 2 = \frac{7}{2}$

2. (a) $(1+X)^n = 1 + {}^nC_1 X + {}^nC_2 X^2 + \ldots$ in general.

$\therefore (1+px)^9 = 1 + {}^9C_1 (px) + {}^9C_2 (px)^2 + \ldots$
$= 1 + 9px + 36p^2x^2 + \ldots$

(b) Now $1 + 9px + 36p^2x^2 = 1 + 36x + qx^2$
$\therefore 9p = 36, \quad p = 4$
$q = 36p^2 = 36 \times 16 = 576$

3. (a)



$AB = \sqrt{5^2 + 1^2} = \sqrt{26}$

(b) Mid-point $\left(\frac{3+4}{2}, \frac{5+0}{2}\right) = \left(3\frac{1}{2}, 2\frac{1}{2}\right)$.

(c) Radius $= \frac{AB}{2} = \frac{\sqrt{26}}{2} \qquad (x-a)^2 + (y-b)^2 = r^2$

$\therefore \left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \left(\frac{\sqrt{26}}{2}\right)^2 = \frac{26}{4} = 6\frac{1}{2}$

4.(a) Geometric series, $a = 120$, $S_\infty = \frac{a}{1-r} = 480$

$\frac{120}{1-r} = 480$, $\frac{120}{480} = 1-r$, $r = 1 - \frac{120}{480} = 3/4$

(b) $u_5 = ar^4 = 120 \times (3/4)^4 = 37.97$

$u_6 = 3/4\, u_5$ so $u_5 - u_6 = u_5(1-3/4) = \frac{1}{4} \times 37.97$
$= 9.49$

(c) $S_n = \frac{a(1-r^n)}{1-r}$, $S_7 = \frac{120(1-0.75^7)}{0.25} = 415.93$ (2d.p.)

(d) nb/ For 4 marks, must show a calculation method !!

$S_n > 300$

$\frac{120(1-0.75^n)}{0.25} > 300$ $\therefore 1 - 0.75^n > \frac{300}{120} \times 0.25$

$1 - 0.75^n > 0.625$

$\therefore 1 - 0.625 > 0.75^n$,

$0.375 > 0.75^n$ i.e. $0.75^n < 0.375$

logs: $n \log 0.75 < \log 0.375$ log 0.75 is negative

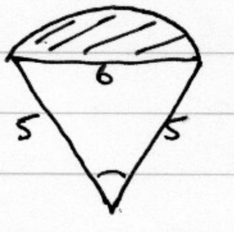
$\therefore n > \frac{\log 0.375}{\log 0.75}$

direction swops.

$n > 3.41$ $\therefore$ minimum $n = 4$.

5.

(a)



Use cosine rule

$6^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \cos AOB$

$\therefore \cos AOB = \frac{25+25-36}{2 \times 25} = \frac{14}{50} = \frac{7}{25}$

b) $AOB = \cos^{-1}\left(\frac{7}{25}\right) = 1.287$

c) Sector area $= \frac{1}{2} r^2 \theta = \frac{1}{2} \times 5^2 \times 1.287 = 16.0875 m^2$

d) shaded area = sector − triangle $= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin\theta = 4.087 m^2$

6. Use ANS button $v = \sqrt{(1.2^t - 1)}$, 0 =, then

$\sqrt{(1.2 \wedge ANS - 1)}$ =, check given values, fill in others

t	0	5	10	15	20	25	30
V	0	1.22	2.28	3.80	6.11	9.72	15.37

6(b) $A \approx \frac{h}{2}\left[y_0+y_n+2(y_1+y_2+\dots)\right]$, $h = 5$ (= x interval).

$\therefore s = \frac{5}{2}\left[0+15.37+2(1.22+2.28+3.80+6.11+9.72)\right] = 154.075$

$= 154\text{m}$ (3 s.figs).

7.(a) $y = 2x^3-5x^2-4x+2$, $dy/dx = 6x^2-10x-4$

(b) At a turning point, $dy/dx = 0$ $\therefore 6x^2-10x-4=0$.

Halve, $3x^2-5x-2=0$, $ac = -6 = -6 \times +1$

$\left(3x+\frac{1}{1}\right)\left(1x-\frac{6}{3}\right) = (3x+1)(x-2) = 0$, $x = -1/3$ or $+2$

Use ANS button! $-1 \div 3 =$, $2 \times \text{ANS}^3 - 5 \times \text{ANS}^2 - 4 \times \text{ANS} + 2$, $y = \frac{73}{27}$.

then $2 =$, repeat, $y = -10$.

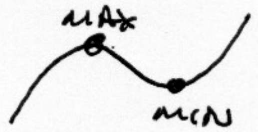
Coordinates $\left(-1/3, 2\frac{19}{27}\right)$, $(2,-10)$.

(c) $d^2y/dx^2 = \frac{d}{dx}(6x^2-10x-4) = 12x-10$

↑ "derivative of..."

(d) At $x = -1/3$, $d^2y/dx^2 = -4-10 = -14$ hence Maximum

At $x = 2$, $d^2y/dx^2 = 24-10 = 14$ hence minimum

(or sketch $+x^3$ cubic, 

and explain).

8.(a) $5\sin(\theta+30) = 3$, $0 \le \theta < 360°$

$\sin(\theta+30) = 3/5$, $\theta+30 = \sin^{-1}(0.6) = 36.9°$, $180-36.9°$

$= 36.9, 143.1°$ $(+n360)$.

$\therefore \theta = 6.9°, 113.1°$

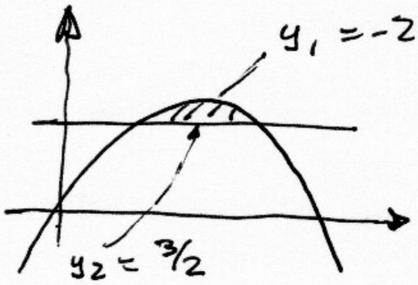
(b)

$\tan^2\theta = 4$ $\therefore \tan\theta = \pm 2$ (all 4 quadrants)

$\theta = \tan^{-1}(2) = 63.4°$, $180-63.4$, $180+63.4$, $360-63.4$

$= 63.4°, 116.6°, 243.4°, 296.6°$

9. (a)



At intersections, $y_1 - y_2 = 0$

$= -2x^2 + 4x - 3/2 = 0$

$\times -2$ $\quad 4x^2 - 8x + 3 = 0$ easier

$ac = 12 = -2 \times -6$

$(2x - \frac{2}{2})(2x - \frac{6}{2})$

$= (2x - 1)(2x - 3) = 0$

$\therefore x = 1/2,\ 3/2$

(b) Area $= \displaystyle\int_{1/2}^{3/2} y_1 - y_2 \, dx = \int_{\frac{1}{2}}^{3/2} -2x^2 + 4x - 3/2 \, dx$

$= \left[-\frac{2}{3}x^3 + 2x^2 - 3/2 x \right]_{1/2}^{3/2}$

$= \left(-2/3(3/2)^3 + 2(3/2)^2 - (3/2)^2\right) - \left(-2/3\left(\frac{1}{2}\right)^3 + 2(1/2)^2 - 3/2(1/2)\right)$

$= 0 - \left(-\frac{1}{12} + \frac{1}{2} - 3/4\right) = 0 - (-1/3) = 1/3$