C2 JAN. 2005

1. $(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots$

$$(3+2x)^5 = 3^5 + {}^5C_1 3^4 (2x) + {}^5C_2 3^3 (2x)^2 + \dots$$
$$= 243 + 5(81)2x + 10(27)(4x^2) + \dots$$
$$= 243 + 810x + 1080x^2 + \dots$$

[Use ${}^nC_r$ button on calculator, or think

${}^nC_r = \dfrac{n!}{(n-r)!\,r!}$ so terms n, $\dfrac{n(n-1)}{2}$, $\dfrac{n(n-1)(n-2)}{2 \times 3}$ etc].

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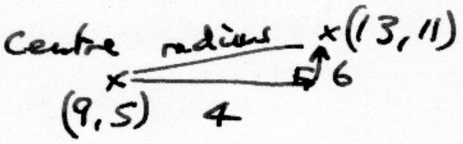
2. A $(5,-1)$ B $(13,11)$.

(a) Mid-point $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right) = \left(\dfrac{18}{2}, \dfrac{10}{2}\right) = (9,5)$

(b) 

Radius $= \sqrt{4^2+6^2} = \sqrt{52}$

General equation of a circle $(x-a)^2 + (y-b)^2 = r^2$

so $(x-9)^2 + (y-5)^2 = 52$

3(a) $3^x = 5$.

Either take $\log_3$ each side (inverse of 3^x)

$\log_3(3^x) = x = \log_3(5) = 1.46$

or logs to any base each side, eg. base 10:

$\log_{10}(3^x) = x \log_{10}(3) = \log_{10}(5)$, $x = \dfrac{\log_{10} 5}{\log_{10} 3} = 1.46$

↑ 3rd rule of logs.

3 (b) $\log_2(2x+1) - \log_2 x = 2$

$= \log_2\left(\frac{2x+1}{x}\right)$.

Do $2^{LHS} = 2^{RHS}$ since 2^x is inverse of $\log_2(x)$

$2^{\log_2\left(\frac{2x+1}{x}\right)} = 2^2$

$= \frac{2x+1}{x} = 4$

$\therefore 2x+1 = 4x$, $1 = 2x$,

$x = \frac{1}{2}$

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4 (a) $5\cos^2 x = 3(1+\sin x)$

Identity $\cos^2 x + \sin^2 x = 1$, $\cos^2 x = 1 - \sin^2 x$

$5(1-\sin^2 x) = 3(1+\sin x)$

$5\sin^2 x + 3\sin x + 3 - 5 = 0$

$5\sin^2 x + 3\sin x - 2 = 0$.

(b) Let $y = \sin x$

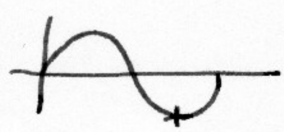
$5y^2 + 3y - 2 = 0$

Think: $5 \times -2 = -10$, factors of -10 adding to 3 are $5, -2$.

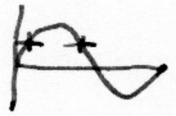
$\left(5y + \frac{-2}{1}\right)\left(y + \frac{5}{5}\right) = (5y-2)(y+1) = 0$

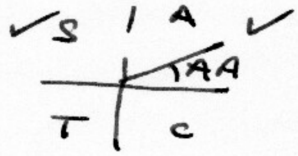
$y = 2/5$ or -1.

At $y = \sin x = -1$, $x = \underline{270^\circ}$



At $y = \sin x = 2/5$, AA $= \sin^{-1}(2/5) = 23.6^\circ$ to 1 d.p.





1st & 2nd quadrants $\Rightarrow$

$x = \underline{23.6^\circ}$ and $180 - 23.6$

$= \underline{156.4^\circ}$

5. $f(x) = x^3 - 2x^2 + ax + b$

(a) Remainder theorem.

"If $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$"

<u>Dividing by $(x-2)$</u> (so $a=2$), $f(2) = 1$.

$$f(2) = 2^3 - 2(2^2) + 2a + b$$
$$= 8 - 8 + 2a + b = 2a + b = 1. \quad — ①$$

<u>Dividing by $(x+1)$</u> (i.e. $a = -1$)

$$f(-1) = (-1)^3 - 2(-1)^2 - a + b$$
$$= -1 - 2 - a + b = 28$$
$$\therefore -a + b = 31 \quad — ②$$

Eliminate b by subtracting ① from ②:

$$-a + b - (2a + b) = 31 - 1 = 30$$
$$= -3a$$
$$\therefore a = -10$$
$$b = 31 + a = 21.$$

(b) Factor theorem: "if $(x-a)$ is a factor, $f(a) = 0$".

~~f(3)~~ $f(x) = x^3 - 2x^2 - 10x + 21$

$$f(3) = 27 - 18 - 30 + 21 = 48 - 48 = 0$$

$\therefore x - 3$ is a factor.

6. Series is $a + ar + ar^2 + ar^3 + \ldots$ (from $u_n = ar^{n-1}$ in formula book).

(a) $\dfrac{4^{th}\text{ term}}{2^{nd}\text{ term}} = \dfrac{ar^3}{ar} = r^2 = \dfrac{5.832}{7.2} = 0.81$

$r = \sqrt{0.81} = 0.9$ <u>since positive</u>.

6 (b) $a = \dfrac{ar}{r} = \dfrac{7.2}{0.9} = 8$

(c) $S_n = \dfrac{a(1-r^n)}{1-r}$ (formula book)

$\therefore S_{50} = \dfrac{8(1-0.9^{50})}{1-0.9} = 79.5877$

$= 79.588$ to 3 d.p.

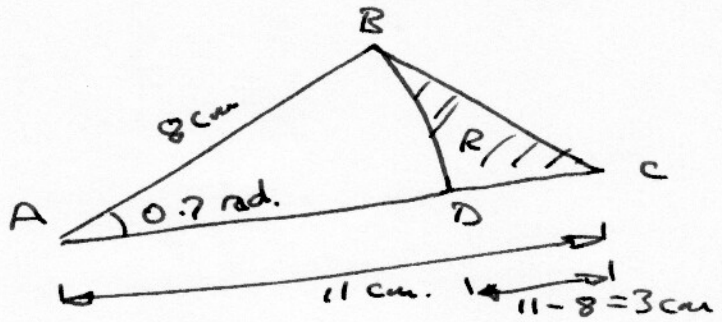
(d) $S_\infty = \dfrac{a}{1-r}$ (formula book)

$= \dfrac{8}{1-0.9} = 80$.

$80 - \text{ANS} = 0.4123$

$= 0.412$ to 3 d.p.

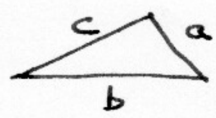
7.



(a) Arc length $= r\theta$ (where θ is in radians)

$= 8 \times 0.7 = 5.6$ cm

(b) Use cosine rule to find length BC ("a")



$a^2 = b^2 + c^2 - 2bc\cos A$.

<u>Check calculator in radians mode!</u>

$a^2 = 64 + 121 - 2(88)\cos 0.7 = 50.388$

$a = 7.0984$ cm

Perimeter BCDB $= 7.0984 + 3 + 5.6 = 15.6984$ cm

$= \underline{15.7\text{ cm to 3 s.f.}}$ Cta

7c. Area of R = area of ABC − area of sector ABD

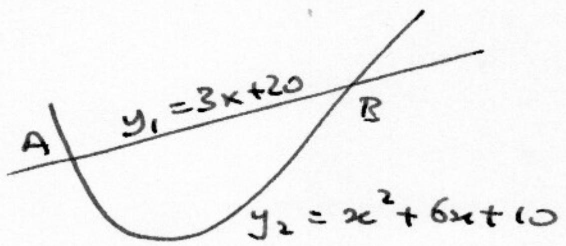
$= \frac{1}{2}(8)(11)\sin 0.7 - \frac{1}{2}(8^2)0.7$

$= \left(\frac{1}{2}ab\sin\theta - \frac{1}{2}r^2\theta\right)$

$= 5.9456 \text{ cm}^2$

$= \underline{5.95 \text{ cm}^2 \text{ to } 3 \text{ s.f.}}$

8.



(a) At A and B, $y_1 = y_2$

$3x + 20 = x^2 + 6x + 10$

$x^2 + 3x - 10 = 0$ $\neq$ "$y_2 - y_1$"

$= (x+5)(x-2)$ so $x = -5$ or 2.

At A, $x = -5$, $y = -15 + 20 = 5$ $\underline{(-5, 5)}$

At B, $x = 2$, $y = 6 + 20 = 26$ $\underline{(2, 26)}$

(b) Area $= \int_{-5}^{2} y_1 - y_2 \, dx = \int_{-5}^{2} -x^2 - 3x + 10 \, dx$ $\times(-1)$

$= \left[-\frac{x^3}{3} - 3\left(\frac{x^2}{2}\right) + 10x\right]_{-5}^{2}$

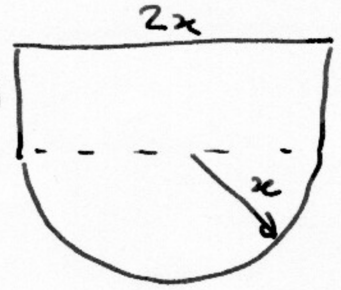
$= \left(-\frac{2^3}{3} - \frac{3}{2}(2^2) + 10(2)\right) - \left(-\frac{(-5)^3}{3} - \frac{3}{2}(-5)^2 + 10(-5)\right)$

$= \frac{-8}{3} - 6 + 20 - \left(\frac{125}{3} - \frac{75}{2} - 50\right)$

$= \frac{-133}{3} + 37\frac{1}{2} + 20 + 50 - 6 = -44\frac{1}{3} + 37\frac{1}{2} + 64$

$= 20 - \frac{1}{3} + 37\frac{1}{2} = 57\frac{1}{6}$

9.



Perimeter $P = 2x + y + y + \frac{2\pi x}{2}$

$= 2x + \cancel{\pi y}\ \pi x + 2y = 80\text{m}$

(a) Area $A = 2xy + \frac{1}{2}\pi x^2$

Since $(2+\pi)x + 2y = 80$, $(1+\pi/2)x + y = 40$,

$y = 40 - (1+\pi/2)x$.

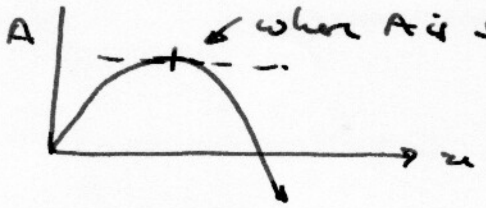
$\therefore A = 2x(40 - (1+\pi/2)x) + \frac{1}{2}\pi x^2$

$= 80x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2$

$= 80x - 2x^2 - \frac{1}{2}\pi x^2$

$= 80x - (2+\pi/2)x^2$

(b)



when A is stationary, $dA/dx = 0$

$\frac{dA}{dx} = 80 - 2(2+\pi/2)x = 80 - (4+\pi)x$

At $\frac{dA}{dx} = 0$, $80 - (4+\pi)x = 0$

$80 = (4+\pi)x$, $x = \frac{80}{4+\pi} = 11.2\text{m}$

(c) If a maximum, $\frac{d^2A}{dx^2}$ will be negative.

$\frac{d^2A}{dx^2} = -(4+\pi)$, negative $\therefore$ proved.

(d) $x = 11.2$, $A = 80x - (2+\pi/2)x^2$

$80 \times ANS - (2+\pi \div 2) * ANS^2 = 448.08\text{m}^2$

$= 448\text{m}^2$ to nearest m^2.