C1 paper F

1. $x^4 = 5x^2 + 14$ $\qquad$ $y^2 = 5y + 14$

Let $y = x^2$ $\qquad$ $y^2 - 5y - 14 = 0$

$(y - 7)(y + 2) = 0$

$y = 7$ or $y = -2$

$x = \pm\sqrt{7}$

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2 $\quad \dfrac{2}{3\sqrt{5} + 7} \dfrac{(7 - 3\sqrt{5})}{(7 - 3\sqrt{5})} = \dfrac{14 - 6\sqrt{5}}{49 - 9 \times 5} = \dfrac{7 - 3\sqrt{5}}{2}$

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3. a) $x^{3/2} = 27$, $x^{1/2} = 3$, $x = 9$

b) $\left(2\tfrac{1}{4}\right)^{-1/2} = \left(\dfrac{9}{4}\right)^{-1/2} = \left(4/9\right)^{1/2} = 2/3$

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4. ~~Touches where gradient = 0~~

$\cancel{\text{ie } 3x^2 + 2ax + b = 0 \text{ at } x = 3}$

$\cancel{\Rightarrow 27 + 6a + b = 0}$

$\cancel{a = -\text{sum of roots} = -(3 + 3 + -1) = -5}$

$\cancel{b = -\Sigma\alpha\beta = -(-3 + -3 + 9) = -3}$

$\cancel{c = -\alpha\beta\gamma = -9}$

$(x+1)(x-3)(x-3) = (x+1)(x^2 - 6x + 9)$

$= x^3 - 5x^2 + 3x + 9$

$(x-\alpha)(x-\beta)(x-\gamma) = (x^2 - (\alpha+\beta)x + \alpha\beta)(x - \gamma)$

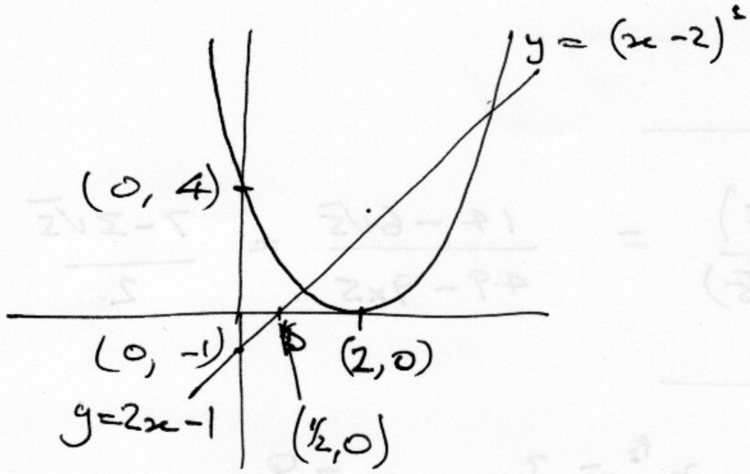
$= x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x - \alpha\beta\gamma$

5. $y = \dfrac{x^4 - 3}{2x^2} = \frac{1}{2}x^2 - \frac{3}{2}x^{-2}$

a) $dy/dx = x + 3x^{-3}$

b) $d^2y/dx^2 = 1 - 9x^{-4}$

6.



Let

$(x-2)^2 = 2x - 1, \quad x^2 - 4x + 4 = 2x - 1$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 1 \text{ or } 5$

so $(x-2)^2 > 2x - 1$ for $x < 1$ ~~and~~ for $x > 5$

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7. $y = x/2 + 3 - 1/x$

(a) $dy/dx = 1/2 + x^{-2}$

At $x = 2$, $dy/dx = 1/2 + 1/4 = 3/4$

(b) At A $x = 2$, $y = 1 + 3 - 1/2 = 3\frac{1}{2}$

$y - 3\frac{1}{2} = 3/4(x - 2) \overset{\times 4}{\Longrightarrow} 4y - 14 = 3x - 6$

$3x - 4y + 8 = 0$

7 (c) Need $dy/dx = 3/4$ again

$\tfrac{1}{2} + x^{-2} = 3/4, \quad x^{-2} = \tfrac{1}{4}, \quad x^2 = 4, \quad x = \pm 2$

Then $y = \dfrac{-2}{2} + 3 - \dfrac{1}{(-2)} = -1 + 3 + \tfrac{1}{2}$

$= 2\tfrac{1}{2}$

B is $(-2, 2\tfrac{1}{2})$.

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8. (a) $y - 3 = 3/2\,(x - 5)$

l_1 is line $y = 3/2\,x + 3 - \dfrac{15}{2} = 3/2\,x - 4\tfrac{1}{2}$

(b) write l_1 as $4y = 6x - 18 \quad (\times 4)$

l_2 is $3x - 4y + 3 = 0$

substitute : $3x - (6x - 18) + 3 = 0$

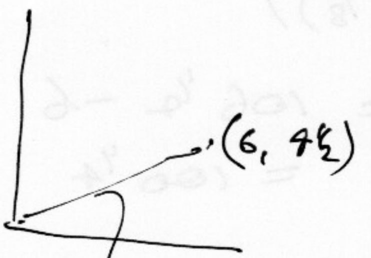
$-3x + 18 + 3 = 0, \quad 21 = 3x, \quad x = 7$

Then $21 + 3 = 4y, \quad y = 6$

B is $(7, 6)$

(c) Mid point is $(6, 4\tfrac{1}{2})$

(d)



l_2 is $4y = 3x + 3$

$y = 3/4\,x + 3/4$

gradient $\dfrac{4\tfrac{1}{2}}{6} = 3/4$

Gradient of l_2 is $3/4$ } same

9. Let $u_n = a + (n-1)d$

(a) then $S_n = \frac{1}{2}n(a+l)$ } from tables

$= \frac{1}{2}n(2a+(n-1)d)$

~~So~~ $u_3 = a + 2d = 5\frac{1}{2}$

$\cancel{S_4 = \frac{1}{2}.4.(a+l) = 22\frac{3}{4}}$

$\cancel{= 2(a+1)}$

$\cancel{\therefore a+1 = 11\frac{3}{8}, \quad a = 10\frac{3}{8}}$

$S_4 = \frac{1}{2}.4(2a+3d) = 4a+6d = 22\frac{3}{4}$

$u_3 = a+2d = 5\frac{1}{2}$ so $4a + 8d = 22$

$\therefore 2d = -\frac{3}{4}, \quad d = -\frac{3}{8}$

$a = 5\frac{1}{2} - 2d = 6\frac{1}{4}$

(b) If $a + (n-1)d = 0$,

$6\frac{1}{4} - (n-1)\frac{3}{8} = 0$

$\times 8 \Rightarrow 50 = 3(n-1)$

$n - 1 = 50/3 = 16\frac{2}{3}, \quad n = 17\frac{2}{3}$

so 17 positive terms

(c) $S_{17} = \frac{1}{2}.17(2\times 6\frac{1}{4} + 16(-\frac{3}{8}))$

$= 17 \times 6\frac{1}{4} - 6 = 106\frac{1}{4} - 6$

$= 100\frac{1}{4}$

10.

(a) ~~At~~ ~~P~~ $(1,1)$ $dy/dx = 8x - 2/x^3$

$= 8 - 2 = 6$

$y - 1 = 6(x-1)$

$y = 6x - 5$

(b) $y = \int dy/dx \, dx = 4x^2 + \frac{1}{x^2} + c$

At $x = 1$, $y = 4 + 1 + c = 5 + c$

Passes through $(1,1)$ so $5 + c = 1$,

$c = -4$

$\therefore \quad y = 4x^2 + \frac{1}{x^2} - 4$

(c) If $4x^2 + 1/x^2 - 4 = 0$

Put $u = x^2 \Rightarrow 4u + \frac{1}{u} - 4 = 0$

$4u^2 - 4u + 1 = 0$

$(2u-1)(2u-1) = 0$, $u = 1/2$

$x = \pm\sqrt{1/2}$

$= \pm \frac{1}{2}\sqrt{2}$