C1 Paper A

1.(a) $\frac{21}{\sqrt{7}} = \frac{21}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{21\sqrt{7}}{7} = 3\sqrt{7}$

(b) $8^{-1/3} = \frac{1}{8^{1/3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

2. $\sum_{r=10}^{30} (7+2r) = (7+2\times10) + (7+2\times11) + (7+2\times12) + \ldots (7+2\times30)$

$= 27 + 29 + 31 + \ldots 67$

If it were $\sum_{r=1}^{30}$, it would be the sum of 30 terms.

As we don't have $r = 1, 2 \ldots 9$, there are $30 - 9 = 21$ terms.

So $a = 27$, $l = 67$, $n = 21$

$S_n = \frac{n}{2}(a+l)$, $S_{21} = \frac{21}{2}(27+67) = \frac{21 \times 94}{2}$

$= 21 \times 47 = 987$

3. Let $y = \frac{6x^2-1}{2\sqrt{x}} = \frac{3x^2 - \frac{1}{2}}{x^{1/2}} = 3x^{1\frac{1}{2}} - \frac{1}{2}x^{1/2}$

[Remember $\sqrt{x} = x^{1/2}$, $\frac{1}{x^a} = x^{-a}$ and/or

$x^a \div x^b = x^{a-b}$]. If $y = x^n$, $dy/dx = nx^{n-1}$.

Then $dy/dx = 3 \times 1\frac{1}{2} x^{1/2} - \frac{1}{2}\left(\frac{1}{2}x^{-1/2}\right) = \frac{9}{2}x^{1/2} - \frac{1}{4}x^{-1/2}$

4. a) $\cancel{3}\ x^2 + 3x > 10$

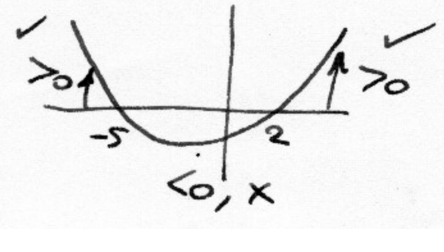
Think, quadratic so make $=0$ to find critical values

$x^2 + 3x - 10 > 0$.

$x^2 + 3x - 10 = (x+5)(x-2)$

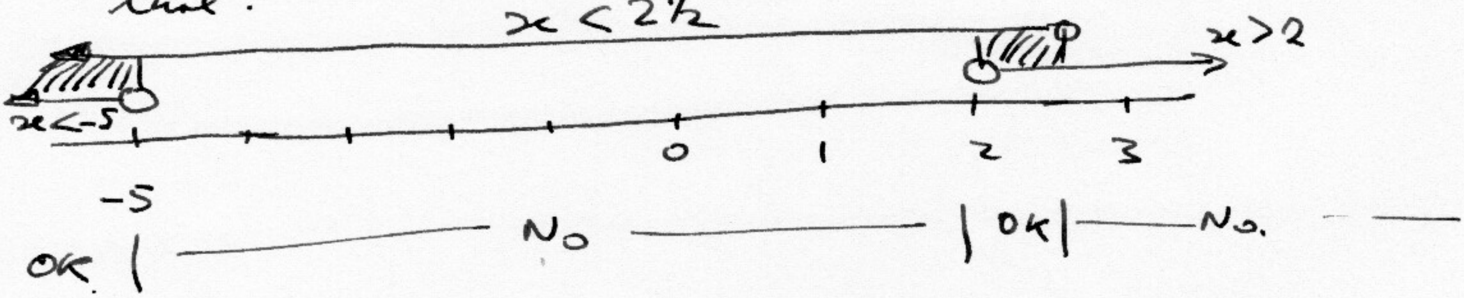
so critical values are $x = -5$, $x = 2$.

$\therefore$ we require $\underline{x < -5 \text{ or } x > 2}$



4 (b) $3x - 2 < x + 3, \Rightarrow 2x < 5, \quad x < 2\tfrac{1}{2}.$

Now draw both solution ranges on a number line:



The lines overlap (i.e. both inequalities are satisfied) for $x < -5$ and $2 < x < 2\tfrac{1}{2}$.

> Remember when writing $<<$ chains, the range must be possible without the x, eg $2 > x > 6$ would be stupid!

5. (a) $u_{n+1} = (u_n)^2 - 1$

$u_1 = k, \quad u_2 = k^2 - 1, \quad u_3 = (k^2-1)^2 - 1$

$= k^4 - 2k^2 + 1 - 1 = k^4 - 2k^2$

(b) $u_2 + u_3 = 11$

$\therefore k^2 - 1 + k^4 - 2k^2 = 11$

> Think: quadratic! Hurray!

> Substitute $y = k^2$

$k^4 - k^2 - 1 - 11 = 0, \quad k^4 - k^2 - 12 = 0$

Let $y = k^2, \quad y^2 - y - 12 = 0$

$= (y-4)(y+3) \Rightarrow \quad y = 4 \text{ or } -3$

$k = \sqrt{y}$, $\sqrt{-3}$ not real so $k = \sqrt{4} = \pm 2$

6(a) $x^2 + 4kx - k = 0$. To "complete the square", write as:

[Need the $-(2k)^2$ to make them equivalent]

$$(x+2k)^2 - (2k)^2 - k = 0$$

$$\therefore\ (x+2k)^2 - 4k^2 - k = 0$$

$$(x+2k)^2 = 4k^2 + k$$

Square root each side. NB. Can only square root the whole side, not bits separately – remember $\sqrt{1+1} \neq \sqrt{1} + \sqrt{1}$!!

$$x + 2k = \pm\sqrt{4k^2+k}$$

$$x = -2k \pm \sqrt{4k^2+k}$$

(b) No real roots if $4k^2 + k$ has no "real" ~~square~~ (ie non-imaginary) square root, ie if $4k^2 + k < 0$.

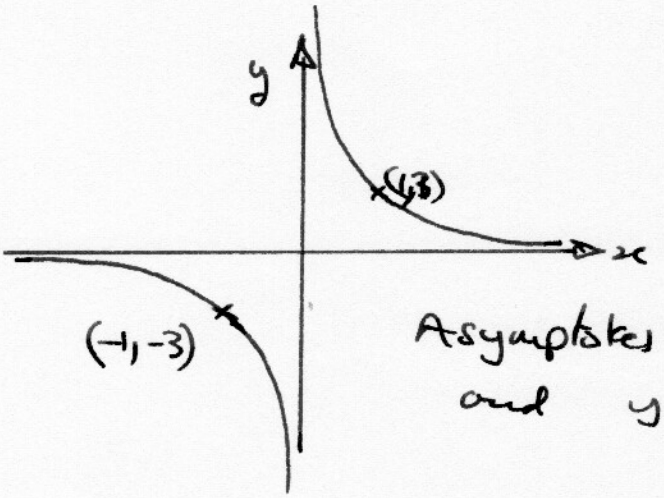
This is a quadratic inequality

- get critical values, $4k^2 + k = 0$, $k(4k+1) = 0$, $k = 0$ or $-1/4$
- sketch

- $\therefore$ No real roots for $-1/4 < k < 0$

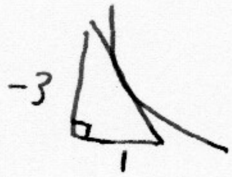
7(a) Mapping $y = 1/x \rightarrow y = 3/x$ needs a stretch $\times 3$ in the y-direction, with $y = 0$ invariant.

(b)



Asymptotes $x = 0$ (y-axis) and $y = 0$ (x-axis)

(c) If the tangent is $y = c - 3x$ (or $y = -3x + c$) its gradient is -3.



$y = 3/x = 3x^{-1}$,

$dy/dx = 3(-x^{-2}) = -3x^{-2}$

When $dy/dx = -3$, $-3x^{-2} = -3$, $x^{-2} = 1$

$\frac{1}{x^2} = 1$, $x^2 = 1$, $x = \pm 1$

At $(1, 3)$, tangent is $y - 3 = -3(x - 1) = -3x + 3$,

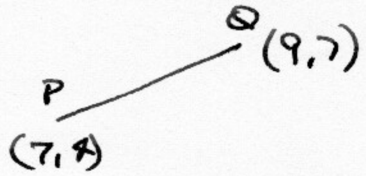
$y = -3x + 6$, $\underline{c = 6}$

At $(-1, -3)$ tangent is $y - (-3) = -3(x - (-1)) = -3(x + 1)$

$y + 3 = -3x - 3$

$y = -3x - 6$, $\underline{c = -6}$

8. (a)



Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-4}{9-7} = \dfrac{3}{2}$

Eq'n of straight line

$y - y_1 = m(x - x_1)$

$y - 4 = 3/2(x-7)$ (or $y - 7 = 3/2(x-9)$)

$= 3/2 x - \frac{21}{2}$

$y = 3/2 x - 10\frac{1}{2} + 4$

$= 3/2 x - 6\frac{1}{2}$. Then $\times 2$,

$2y = 3x - 13$.

$\underline{3x - 2y - 13 = 0}$

(b) Gradient $m = 8$ through $(0,0)$, $\underline{y = 8x}$

(c) Line intersection $\Rightarrow$ simultaneous equations

$\left\{ \begin{array}{l} y = 8x \text{ substitute} \\ 3x - 2y - 13 = 0 \end{array} \right.$

$3x - 2(8x) - 13 = 3x - 16x - 13$

$= -13x - 13 = 0$

$-13x = 13$, $x = \dfrac{13}{-13} = -1$, $y = 8x = -8$.

So R is $(-1,-8)$, P is $(7,4)$ from question.

Pythagoras: $\left. \begin{array}{l} OR = \sqrt{1^2 + 8^2} = \sqrt{65} \\ OP = \sqrt{7^2 + 4^2} = \sqrt{65} \end{array} \right\}$ Same ✓.

9. (a) Remember if $y = f(x)$, $f'(x)$ means dy/dx.

$\Rightarrow$ integrate to find y.

$$y = \int dy/dx \, dx = \int 6 - 4x - 3x^2 \, dx = 6x - 4\left(\frac{x^2}{2}\right) - 3\left(\frac{x^3}{3}\right) + c$$

$$= 6x - 2x^2 - x^3 + c.$$

At the origin, $x = 0$ and

$$y = 6(0) - 2(0^2) - 0^3 + c = c = 0$$

$$\therefore y = 6x - 2x^2 - x^3$$

(b) A, B and $x = 0$ are the roots of $6x - 2x^2 - x^3 = 0$.

Factorise: $-x(x^2 + 2x - 6) = 0$.

$x^2 + 2x - 6 = 0$ does not easily factorise, but does have roots (~~think~~ $b^2 - 4ac = 2^2 + 4 \times 1 \times 6 = 28$), so use the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 + 24}}{2} = -1 \pm \frac{\sqrt{28}}{2} = -1 \pm \frac{\sqrt{4}\sqrt{7}}{2}$$

$$= -1 \pm \sqrt{7}.$$

Distance AB $= (-1 + \sqrt{7}) - (-1 - \sqrt{7}) = -1 + 1 + \sqrt{7} + \sqrt{7} = \underline{\underline{2\sqrt{7}}}$

($k = 2$).

10 (a) $y = x + 3/x = x + 3x^{-1}$, $dy/dx = 1 + 3(-x^{-2}) = 1 - 3/x^2$

At P, $x = 1$, $dy/dx = 1 - 3 = -2$.

(b) Gradient of normal $= \frac{-1}{-2} = 1/2$

(c) At P, $y = 1 + 3/1 = 4$.

Normal $y - 4 = 1/2(x - 1) = 1/2 x - 1/2$, $y = 1/2 x + 3\frac{1}{2}$

At intersection of normal + curve, $x + 3/x = 1/2 x + 3\frac{1}{2}$

($\times x$) $x^2 + 3 = 1/2 x^2 + 3\frac{1}{2}x$, $1/2 x^2 - 3\frac{1}{2}x + 3 = 0$, $x^2 - 7x + 6 = 0$,

$(x - 6)(x - 1) = 0$, $x = 1$ or 6. P is at $x = 1$, new point

is $(6, 6 + 3/6) = \underline{(6, 6\frac{1}{2})}$