C1 MAY 2010

1. $\sqrt{75} - \sqrt{27} = \sqrt{25\times 3} - \sqrt{9\times 3} = 5\sqrt{3} - 3\sqrt{3} = 2\sqrt{3}$

2. $\int 8x^3 + 6x^{1/2} - 5\,dx = 8\left(\frac{x^4}{4}\right) + 6\,\frac{x^{3/2}}{(3/2)} - 5x + c$

$= 2x^4 + 4x^{3/2} - 5x + c$

3.(a) $3(x-2) < 8 - 2x$

$3x - 6 < 8 - 2x$

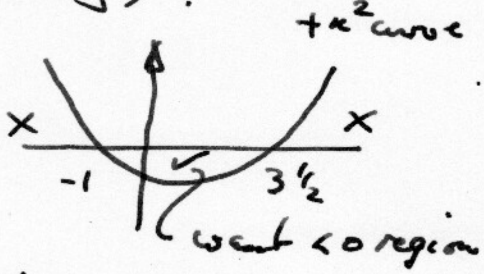
$5x < 14, \quad x < \frac{14}{5}$ or $x < 2\frac{4}{5}$

(b) $(2x-7)(x+1) < 0$. (Quadratic inequality)

Critical values:

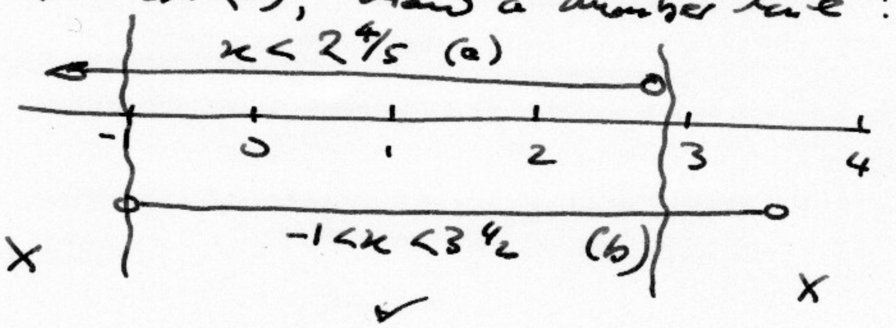
$2x - 7 = 0, \quad x = 3\frac{1}{2}$
$x + 1 = 0, \quad x = -1$

Now sketch:



$\boxed{-1 < x < 3\frac{1}{2}}$ – check is a valid chain, $\ldots < x < \ldots$

(c) Both (a) and (b), draw a number line:

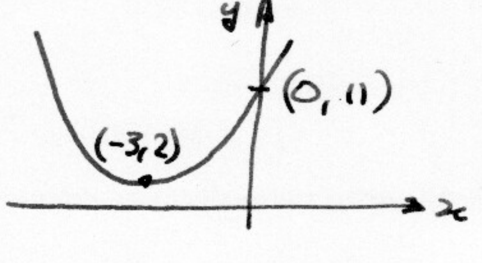


Need $-1 < x < 2\frac{4}{5}$

4.(a) Completing the square. $x^2 + 6x + 11 = (x+3)^2 - 9 + 11$

$= (x+3)^2 + 2$

(b) Minimum is $y = 2$ at $x = -3$



(c) $b^2 - 4ac = 6^2 - 4\times 1\times 11$

$= 36 - 44 = -8$

$\therefore$ No real roots, does not cut x-axis.

5. Recurrence relation, $a_{n+1} = \sqrt{a_n^2 + 3}$

(a) $a_2 = \sqrt{a_1^2+3} = \sqrt{2^2+3} = \sqrt{7}$

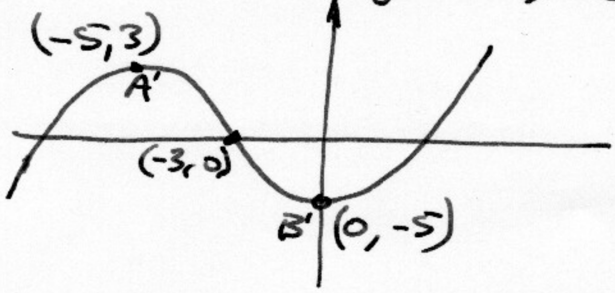
$a_3 = \sqrt{a_2^2+3} = \sqrt{7+3} = \sqrt{10}$

(b) $a_4 = \sqrt{10+3} = \sqrt{13}$

$a_5 = \sqrt{13+3} = \sqrt{16} = 4$

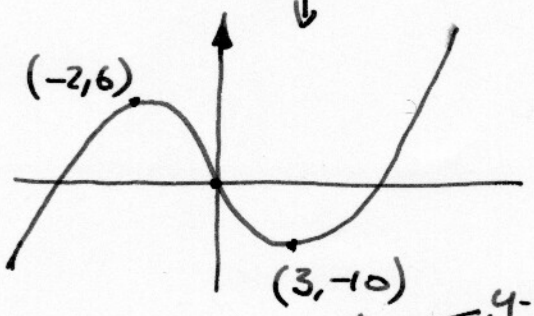
6. Curve transformations.

(a) $y = f(x+3)$ is $y = f(x)$ curve moved $\xleftarrow{3}$



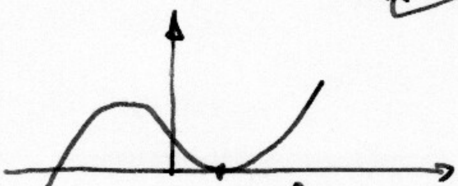
check: $f(3) = -5$ from fig. 1.
At $x = 0$,
$f(x+3) = f(3) = -5$ ✓

(b) $y = 2f(x)$ is $y = f(x)$ stretched $\updownarrow$ $\times 2$.



y-values double.

(c)



(3,0) ↑ moved up 5, $\boxed{a = 5}$

7. $y = 8x^3 - 4\sqrt{x} + \dfrac{3x^2+2}{x}$

"$x>0$" to avoid having $\sqrt{\text{negative number}}$ or $\div 0$

* Must write as a series of x^n terms:

$y = 8x^3 - 4x^{1/2} + 3x + 2x^{-1}$

$dy/dx = 24x^2 - 2x^{-1/2} + 3 - 2x^{-2}$

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8. (a)

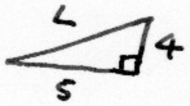
gradient $m = \dfrac{4-0}{7-2} = \dfrac{4}{5}$

$y - y_1 = m(x - x_1)$, $\quad y - 0 = 4/5(x-2)$

(x5): $5y = 4(x-2) = 4x - 8$

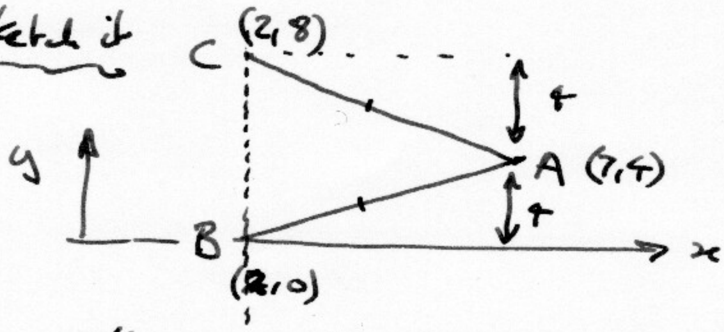
$\therefore 4x - 5y - 8 = 0$

(b) Pythagoras



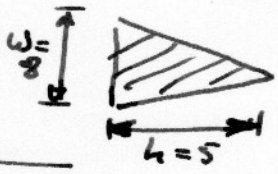
$L = \sqrt{5^2+4^2} = \sqrt{25+16} = \sqrt{41}$

(c) Sketch it



$\boxed{t = 8}$

(d) Area $= 1/2\, wh = \frac{1}{2} \times 8 \times 5 = 20$



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9. Arithmetic series (constant d).

(a) $n = 30$, $u_n = a + (n-1)d \quad \therefore \underline{u_{30} = £40.75 = a + 29d}$

(b) $S_n = \frac{n}{2}(a + l)$ from formula book

$\therefore S_{30} = \frac{30}{2}(a + 40.75) = 15(a + 40.75) = 1005$

(c) $15\overline{)1005}$ = 67 ($1005 - 900 = 105$, $105 - 105$)

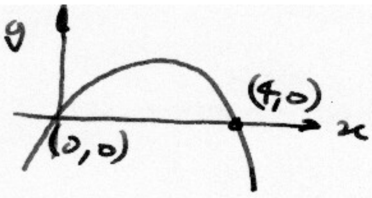
$a + 40.75 = 67 \quad \therefore \boxed{a = 26.25}$

$a + 29d = 26.25 + 29d = 40.75$

$\therefore 29d = 14.5, \quad \boxed{d = 0.5}$

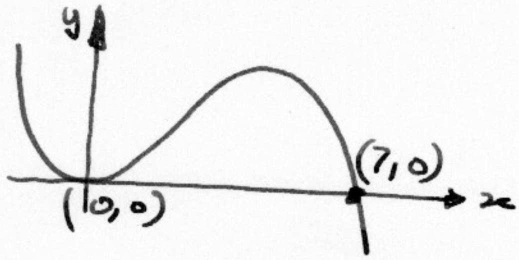
10 (a) (i) $y = x(4-x)$ $\rightarrow$ roots $x=0$, $x=4$

$= -x^2 + 4x$ $\rightarrow$ $-x^2$ quadratic



(ii) $y = x^2(7-x)$ $\rightarrow$ roots $x=0$ (repeated), $x=7$

$= -x^3 + 7x^2$ $\rightarrow$ $-x^3$ cubic

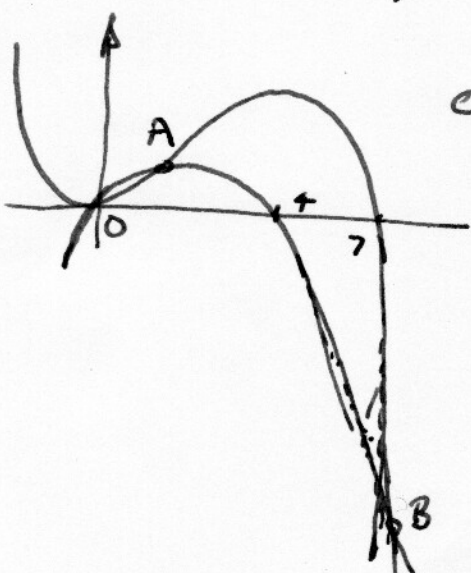


(b) $\left.\begin{array}{l} y = -x^2 + 4x \\ y = -x^3 + 7x^2 \end{array}\right\}$ simultaneous equations, subtract to eliminate y.

$(-x^2 + 4x) - (-x^3 + 7x^2) = y - y = 0$

$x^3 - 8x^2 + 4x = 0$, $\quad x(x^2 - 8x + 4) = 0$

(c)



Curves intersect at O, A & B.

Solve $x^2 - 8x + 4 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{8 \pm \sqrt{64-16}}{2}$

$= 4 \pm \dfrac{\sqrt{48}}{2} = 4 \pm 2\sqrt{3}$

$\therefore$ A at $x = 4 - 2\sqrt{3}$, B at $x = 4 + 2\sqrt{3}$

$y = -x^2 + 4x = -(16 - 16\sqrt{3} + 12) + 16 - 8\sqrt{3}$

$= -12 + 8\sqrt{3}$

$\therefore$ A is $(4 - 2\sqrt{3},\ -12 + 8\sqrt{3})$

11. a) $dy/dx = 3x - \dfrac{5}{\sqrt{x}} - 2 = 3x - 5x^{-1/2} - 2$

$y = \int dy/dx \, dx = 3\left(\dfrac{x^2}{2}\right) - 5\left(\dfrac{x^{1/2}}{(1/2)}\right) - 2x + c = \frac{3}{2}x^2 - 10x^{1/2} - 2x + c$

At $x=4$, $y = \frac{3}{2}(16) - 10(2) - 8 + c = -4 + c = 5 \therefore c = 9$

$y = \frac{3}{2}x^2 - 10x^{1/2} - 2x + 9$

b) At $x=4$, $dy/dx = 3(4) - \frac{5}{2} - 2 = 7\frac{1}{2}$

Tangent $y - 5 = \frac{15}{2}(x-4)$, $\quad 2y - 10 = 15(x-4) = 15x - 60$

$15x - 2y - 50 = 0$