MAY 2006 C1

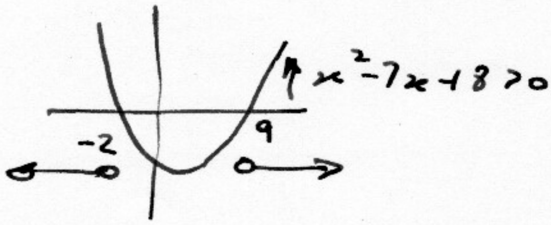
1. $\int 6x^2+2x+x^{-1/2}\,dx = 6\left(\frac{x^3}{3}\right)+2\left(\frac{x^2}{2}\right)+\frac{x^{1/2}}{(1/2)}+c$

$= 2x^3+x^2+2x^{1/2}+c$

2. $x^2-7x-18>0$.

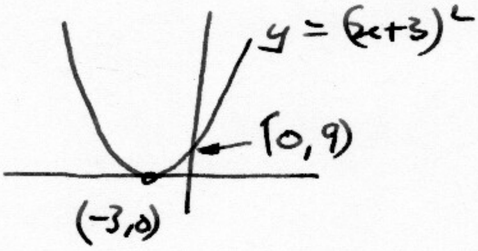
Critical values, set $x^2-7x-18=0$

$= \cancel{(x-6)} (x-9)(x+2)$, $x=-2$ or 9.

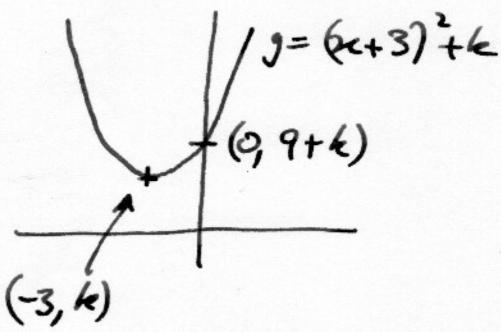


$\underline{x<-2 \text{ or } x>9}$

3.(a) $y=(x+3)^2$, graph of $y=x^2$ translated 3 to left



(b) Same but shifted up by k



4. (a) $a_{n+1} = 3a_n - 5$ so $a_2 = 3a_1 - 5$

$= 3\times 3 - 5 = 9 - 5 = 4$

$a_3 = 3\times a_2 - 5 = 12 - 5 = 7$

(b) $\sum_{r=1}^{5} a_r = a_1 + a_2 + a_3 + a_4 + a_5$

$a_4 = 3\times a_3 - 5 = 21 - 5 = 16$

$a_5 = 3\times a_4 - 5 = 43.$

$\therefore \sum_{r=1}^{5} a_r = 3 + 4 + 7 + 16 + 43 = 73$

5 (a) $\frac{d}{dx}(x^4 + 6x^{1/2}) = 4x^3 + 6(\frac{1}{2}x^{-1/2})$

$= 4x^3 + 3x^{-1/2}$

Remember, $\sqrt[n]{x} = x^{1/n}$, $\sqrt{x} = x^{1/2}$, $\frac{d}{dx}(x^n) = nx^{n-1}$

(b) $\frac{(x+4)^2}{x} = \frac{x^2 + 8x + 16}{x} = x + 8 + 16x^{-1}$

$\frac{d}{dx}(x + 8 + 16x^{-1}) = 1 + 0 + 16(-x^{-2})$

$= 1 - 16x^{-2}$

6 (a) $(4+\sqrt{3})(4-\sqrt{3}) = 16 - 4\sqrt{3} + 4\sqrt{3} - (\sqrt{3})^2$

$= 16 - 3 = 13$

(b) $\frac{26}{4+\sqrt{3}} = \frac{26}{(4+\sqrt{3})}\left(\frac{4-\sqrt{3}}{4-\sqrt{3}}\right) = \frac{26(4-\sqrt{3})}{13} = 2(4-\sqrt{3})$

$= 8 - 2\sqrt{3}$

7. We're told last term $l = 9$ and $S_{11} = 77$ km.

Use $S_n = n/2\,(a + l)$, $S_{11} = \frac{11}{2}(a+9) = 77$

$\div 11$ $7 = \frac{1}{2}(a+9)$, $a + 9 = 14$, $a = 14 - 9 = 5$ km.

Then 11^{th} term $= a + (n-1)d$

$= 5 + 10d = 9$,

$10d = 9 - 5 = 4$, $d = 0.4$ km

[or use $S_n = n/2\,(2a + (n-1)d)$ and solve simultaneous equations].

8. (a) Equal roots $\Rightarrow$ discriminant $b^2 - 4ac = 0$

$\Rightarrow (2p)^2 - 4 \times 1 \times (3p+4) = 0$

$4p^2 - 12p - 16 = 0$

$\div 4$ $p^2 - 3p - 4 = 0$

$= (p+1)(p-4)$, $p = -1$ or $+4$.

Since p is a positive constant, $\underline{p = 4}$.

(b) $x^2 + 2px + (3p+4)$ with $p = 4$

$= x^2 + 8x + 16 = 0$

$= (x+4)^2$, $\underline{x = -4}$

↑repeated root!

9. (a) $f(x) = (x^2 - 6x)(x-2) + 3x$

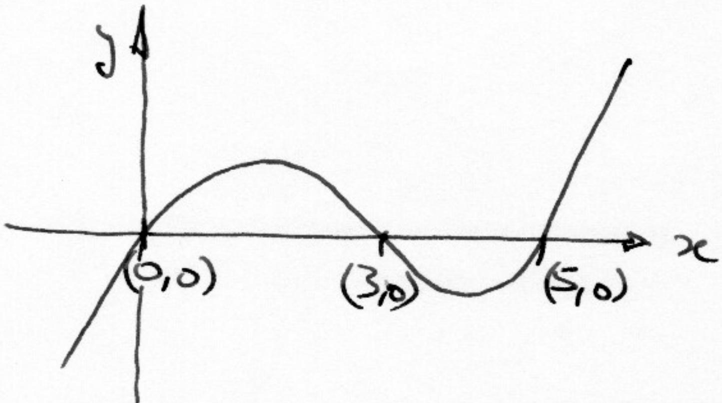
$= x^3 - 6x^2 - 2x^2 + 12x + 3x$

$= x^3 - 8x^2 + 15x = x(x^2 - 8x + 15)$

(b) $f(x) = x(x-3)(x-5)$

9(c) Cubic with $+x^3$ so  or

Cuts x-axis in 3 places, $x = 0$, 3 and 5



10. $f'(x) = 2x + 3x^{-2}$

remember $\frac{1}{x^a} = x^{-a}$,
just like $\frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

(a) $f(x) = \int f'(x)\,dx$

$$= 2\left(\frac{x^2}{2}\right) + 3\left(\frac{x^{-1}}{-1}\right) + c = x^2 - 3x^{-1} + c$$

At $x = 3$, $y = 7\frac{1}{2}$ so

$$3^2 - \frac{3}{3} + c = 9 - 1 + c = 8 + c = 7\frac{1}{2},$$

$$c = 7\frac{1}{2} - 8 = -\frac{1}{2}$$

$\therefore f(x) = x^2 - 3x^{-1} - \frac{1}{2}$

(b) $f(-2) = \cancel{(-2)^2 - \frac{3}{-2}}\ (-2)^2 - \left(\frac{3}{-2}\right) - \frac{1}{2} = 4 + 1\frac{1}{2} - \frac{1}{2} = 5$

(c) With $y = f(x)$, $dy/dx = f'(x) = 2x + 3x^{-2}$

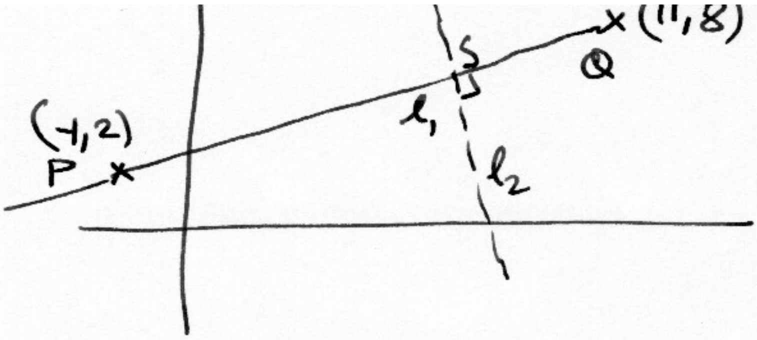
At $x = -2$, $dy/dx = -4 + \frac{3}{(-2)^2} = -4 + \frac{3}{4} = -3\frac{1}{4}$

$y - 5 = -3\frac{1}{4}(x - -2) = \frac{-13}{4}(x + 2)$

$(\times 4)$ $4y - 20 = -13x - 26$

$13x + 4y + 6 = 0$

11.



(a)

Gradient of PQ is $\frac{y_2-y_1}{x_2-x_1} = \frac{8-2}{11--1} = \frac{6}{12} = \frac{1}{2}$

l_1 is $y-2 = \frac{1}{2}(x-(-1)) = \frac{1}{2}x + \frac{1}{2}$

$\underline{y = \frac{1}{2}x + 2\frac{1}{2}}$

(b) $l_2 \perp$ to l_1, gradient $m_2 = \frac{-1}{(\frac{1}{2})} = -2$

$y - y_1 = m(x - x_1)$,

$y - 0 = -2(x-10) = -2x + 20$

Find where $y = -2x + 20$ intersects $y = \frac{1}{2}x + 2\frac{1}{2}$.

$\Rightarrow$ Substitute for y,

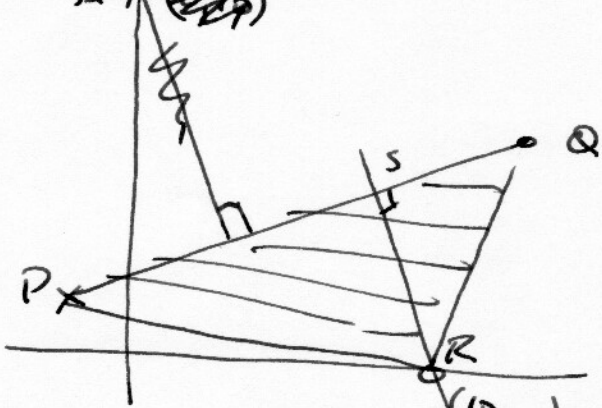
$-2x + 20 = \frac{1}{2}x + 2\frac{1}{2}$, $20 - 2\frac{1}{2} = 2\frac{1}{2}x$
$= 17\frac{1}{2}$,

$5x = 35$, $x = 7$

$y = -2x + 20 = -14 + 20 = 6$

S is at $(7,6)$.

(c)



PQ = base,
SR = height
(since $l_1 \perp l_2$).

$|PQ| = \sqrt{(11--1)^2 + (8-2)^2}$
$= \sqrt{12^2 + 6^2} = \sqrt{144+36} = \sqrt{180}$
$= \sqrt{5 \times 36} = 6\sqrt{5}$

$|RS| = \sqrt{(10-7)^2 + (0-6)^2}$
$= \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$.

Area $= \frac{1}{2} \times 3\sqrt{5} \times 6\sqrt{5} = \frac{1}{2} \times 18 \times 5 = 9 \times 5 = \underline{45}$