JUNE 2009 C1

1 (a) $(3\sqrt{7})^2 = 9 \times 7 = 63$ $\quad\{$ Think $(xy)^n = x^n y^n \}$

(b) $(8+\sqrt{5})(2-\sqrt{5}) = 16 + 2\sqrt{5} - 8\sqrt{5} - 5 = 11 - 6\sqrt{5}$.

2. $32\sqrt{2} = 2^5 \times 2^{1/2} = 2^{5\frac{1}{2}}, \quad a = 5\frac{1}{2}$.

3. $y = 2x^3 + \dfrac{3}{x^2} = 2x^3 + 3x^{-2}$

(a) $dy/dx = 6x^2 - 6x^{-3}$

(b) $\displaystyle\int y\,dx = \frac{2x^4}{4} + 3\frac{x^{-1}}{-1} + c = \frac{1}{2}x^4 - 3x^{-1} + c$

4. (a) $4x - 3 > 7 - x$

$\therefore \quad 5x > 10$

$x > 2$

(b). Quadratic inequality.

$2x^2 - 5x - 12 < 0$.

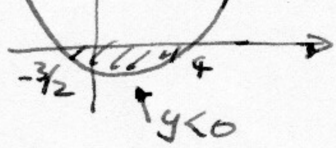
Set $= 0$ to find critical values: $2x^2 - 5x - 12 = 0$

$ac = 2 \times -12 = -24 = -8 \times 3, \quad -8 + 3 = -5$.

$2x^2 - 8x + 3x - 12 = 2x(x-4) + 3(x-4) = 0$

$(2x+3)(x-4) = 0, \quad x = -3/2$ or 4

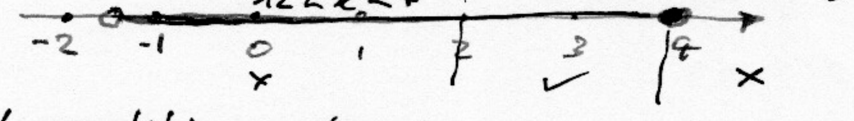
Now either sketch



$\underline{-3/2 < x < 4}$

or try a value, eg $x = 0$, $-12 < 0$ ✓ so need region <u>between</u> the critical values.

(c)



Inequalities overlap for $2 < x < 4$

5. Define the n^{th} term as being in ~~year~~ $n = \text{year} - 1950$ (so $n = 1$ in 1951 etc).

In 1960, $n = 1960 - 1950 = 10$,

$$u_{10} = 2400 = a + (n-1)d = a + 9d.$$

In 1990, $u_{40} = 600 = a + 39d$

$$(a+39d) - (a+9d) = 30d = 600 - 2400 = -1800$$

(a) $\therefore d = -60$

$$a + 9d = a - 540 = 2400$$

(b) $\therefore a = 2940$

(c) $S_n = n/2\,(a + l)$

$$S_{40} = \frac{40}{2}(2940 + 600) = 20 \times 3540 = 70800$$

6. "Equal roots" means the discriminant $b^2 - 4ac = 0$

$x^2 + 3px + p = 0 \qquad a = 1,\ b = 3p,\ c = p$

$b^2 - 4ac = (3p)^2 - 4p = 9p^2 - 4p = 0$

$p(9p - 4) = 0, \quad p = 0$ or $4/9$

But p is 'non-zero' $\therefore p = \frac{4}{9}$

7. $a_1 = k \qquad a_{n+1} = 2a_n - 7$

(a) $\therefore a_2 = 2a_1 - 7 = 2k - 7$

(b) $a_3 = 2a_2 - 7 = 2(2k-7) - 7 = 4k - 14 - 7 = 4k - 21$

(c) $\sum_{r=1}^{4} a_r$ means $a_1 + a_2 + a_3 + a_4$

$a_4 = 2a_3 - 7 = 8k - 42 - 7 = 8k - 49$

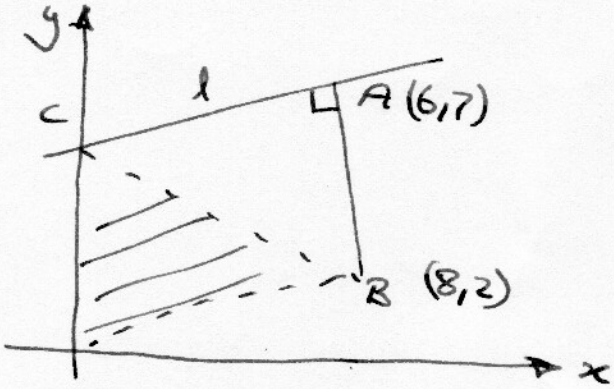
$\therefore \sum_1^4 a_r = k + (2k-7) + (4k-21) + (8k-49)$

$= 15k - 77 = 43$

$15k = 120, \quad \underline{k = 8}$

8.

(a)



Gradient of AB is

$m_{ab} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-2}{6-8} = \frac{5}{-2}$

$\therefore$ gradient of l is $\frac{-1}{(-5/2)}$

$= 2/5.$

Line l is $y - y_1 = m(x - x_1)$

$y - 7 = 2/5(x-6)$

($\times 5$) $5y - 35 = 2(x-6) = 2x - 12$

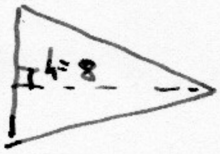
$\therefore \quad 2x - 5y + 23 = 0$

(b) At $x = 0$, $-5y + 23 = 0$, $y = \frac{23}{5}$.

[check: < 7, below A - looks right].

(c)

$w = 23/5$



area $= \frac{1}{2} wh = 1/2 \times \frac{23}{5} \times 8$

$= \frac{92}{5} = 18\frac{2}{5}$ units^2.

9. (a) $f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}} = \frac{3^2 - 24\sqrt{x} + 16x}{\sqrt{x}} = 9x^{-1/2} - 24 + 16x^{\frac{1}{2}}$

$\therefore A = 16, B = -24$

(b) $f'(x) = -\frac{9}{2} x^{-3/2} + 8x^{-1/2}$

(c) At $x = 9$, $x^{1/2} = 3$, $x^{-1/2} = \frac{1}{3}$, $x^{-3/2} = (\frac{1}{3})^3 = \frac{1}{27}$

$\therefore f'(9) = -\frac{9}{2}(\frac{1}{27}) + \frac{8}{3} = -\frac{1}{2}(\frac{1}{3}) + 8(\frac{1}{3})$

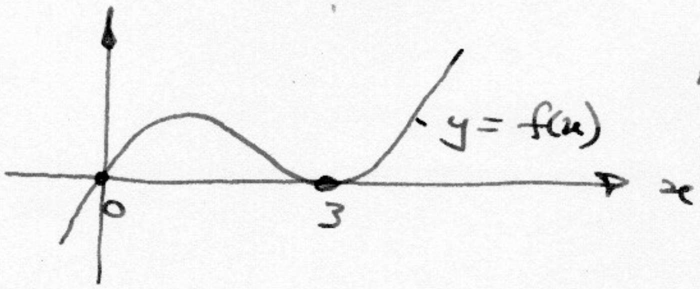
$= \frac{7\frac{1}{2}}{3} = 2\frac{1}{2}$

10. (a) $x^3 - 6x^2 + 9x = x(x^2 - 6x + 9)$

$= x(x-3)^2$

(b). At $y = 0$, $x = 0$ or 3.

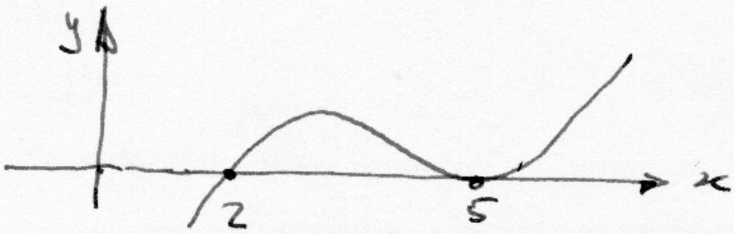
The $x = 3$ point is a repeated root – the curve just touches the x-axis here.



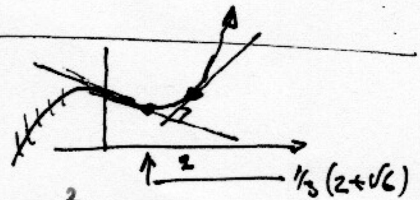
A "$+x^3$" cubic.

(c) $y = (x-2)^3 + 6(x-2)^2 + 9(x-2) = f(x-2)$,

the curve from (b) translated $\xrightarrow{2}$.



11. $y = x^3 - 2x^2 - x + 9$



(a) At $x = 2$, on curve C $y = 2^3 - 2\times2^2 - 2 + 9$

$= 8 - 8 + 9 - 2 = 7$

$\therefore$ (2, 7) lies on C.

(b) $dy/dx = 3x^2 - 4x - 1$, at $x = 2$, $dy/dx = 12 - 8 - 1 = 3$

$\therefore$ Tangent is $y - 7 = 3(x-2) = 3x - 6$, $\underline{y = 3x + 1}$.

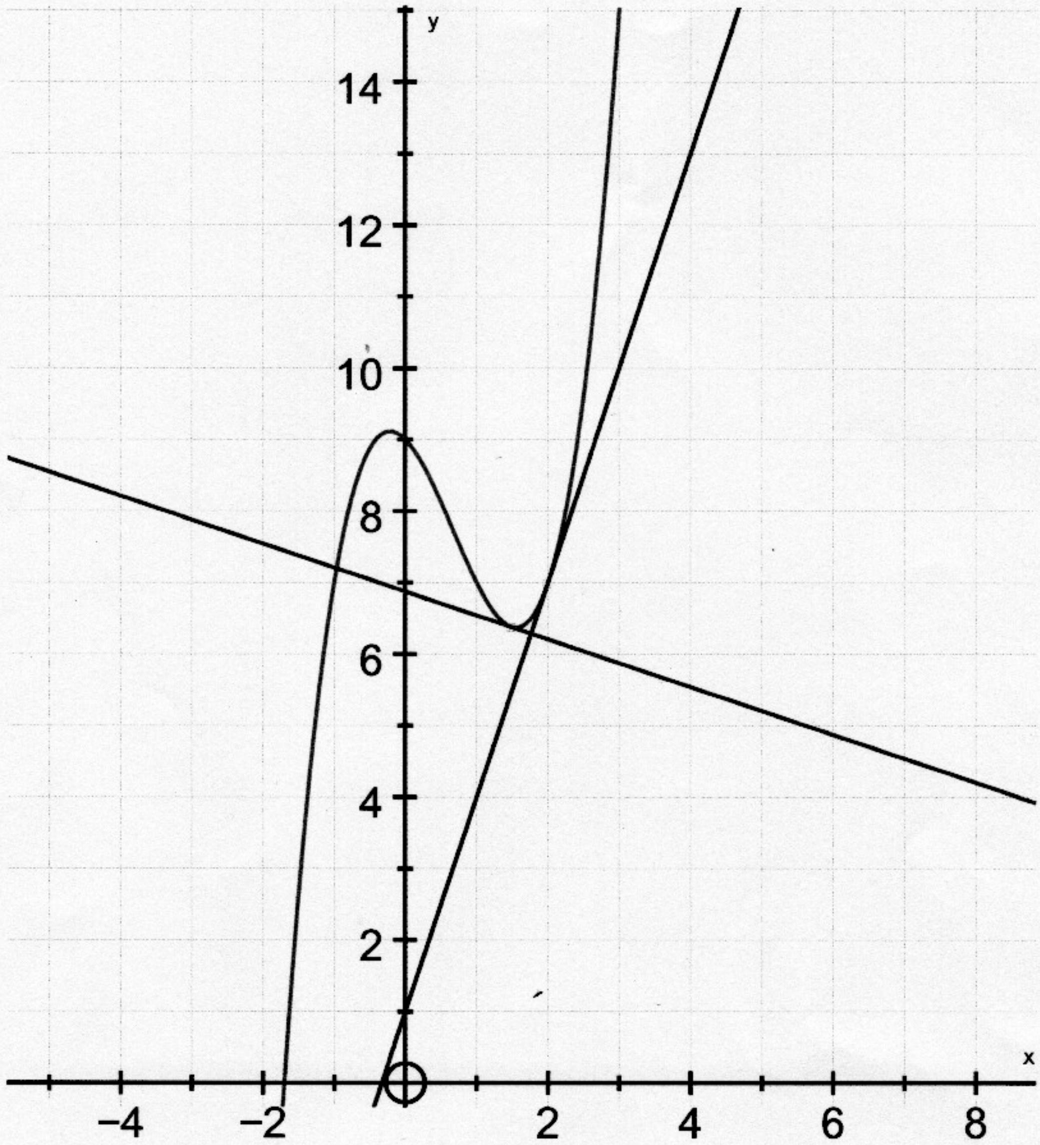
(c) Gradient of tangent at Q is $-\frac{1}{3}$ ($\perp$ to gradient $= 3$).

$\therefore dy/dx = -1/3$, $3x^2 - 4x - 1 = -1/3$, $3x^2 - 4x - 2/3 = 0$

(×3) $9x^2 - 12x - 2 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{144 + 72}}{18} = \frac{2}{3} \pm \frac{\sqrt{36\times6}}{18} = 1/3(2 \pm \sqrt{6})$

y is only defined for $\underline{x > 0}$ $\therefore$ Need positive root, $x = \frac{1}{3}(2 + \sqrt{6})$.



y
14
12
10
8
6
4
2
x
−4
−2
2
4
6
8