# C1 June 2008

1. $\int 2+5x^2\,dx$

$= 2x + 5\left(\frac{x^3}{3}\right) + c$

$= 2x + \frac{5}{3}x^3 + c$

Remember $\frac{d}{dx}(x^n) = nx^{n-1}$,

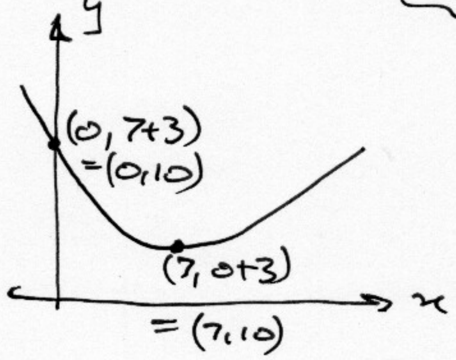
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

2. $x^3 - 9x = x(x^2-9) = x(x-3)(x+3)$

"difference of two squares"

3. (a) $y = f(x)+3$

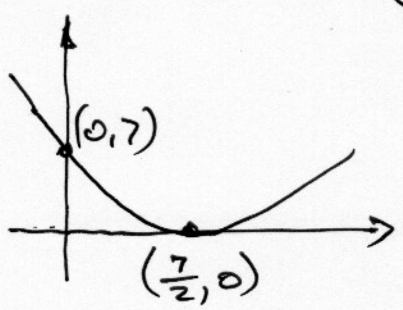
"Whatever y-value $f(x)$ gives you, add 3 to it"
So translates up 3




(b)

$y = f(2x)$

"Doubling the frequency."
To get your y-values, need to start with half the original $x$.
So stretches ↔ by $\times\frac{1}{2}$




Check:
here $x = 7/2$,
$2x = 7$, $f(2x) = f(7) = 0$.

4. (a) ~~f(x)~~ $f(x) = 3x + x^3$

so $f'(x) = 3 + 3x^2$

"$x>0$" simply means this function "$f$" is only defined (should only be used with) positive $x$ values..

4 (b) From (a), $f'(x) = 3 + 3x^2$.

Where $f'(x) = 15$, $3 + 3x^2 = 15$

$\therefore 3x^2 = 12$, $x^2 = 4$, $x = \pm\sqrt{4} = \pm 2$.

But, $x$ is $> 0$ $\therefore x = 2$

5. (a). $x_2 = ax_1 - 3$

$\cancel{x_2 = ax}$

$x_1 = 1$ so

$x_2 = a - 3$.

$\boxed{\text{$x_{n+1} = ax_n - 3$ is a recurrence relationship meaning "to find the next value, multiply by $a$, add 3".}}$

(b) $x_3 = ax_2 - 3$

$= a(a-3) - 3 = a^2 - 3a - 3$

(c) $x_3 = 7$

$\therefore a^2 - 3a - 3 = 7$,

$a^2 - 3a - 10 = 0$

$= (a-5)(a+2)$ $\therefore a = -2$ or $5$.

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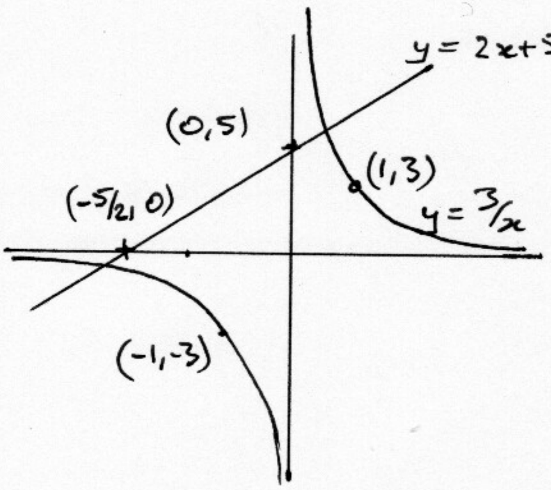
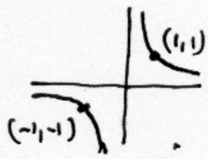
6. (a)




You know $y = \frac{1}{x}$ is a curve




$y = 3/x$ is a stretched $\updownarrow \times 3$ version of this

Think $\frac{3}{0} = \infty$, the curve goes to infinity at $x = 0$

(b) [Line intersections are simultaneous equations].

$y = 2x + 5$ and $y = 3/x \Rightarrow 2x + 5 = 3/x$

$(\times x)$: $2x^2 + 5x = 3$, $2x^2 + 5x - 3 = 0$

$\boxed{\begin{aligned} ac &= 2 \times -3 = -6 \\ &= 6 \times -1, \quad 6 - 1 = 5 = b \end{aligned}}$

$(2x - \frac{1}{1})(1x + \frac{6}{2}) = (2x-1)(x+3)$, $x = -3$ or $x = 1/2$.

Sub into $y = 2x + 5$: points are $(-3, -1)$ and $(1/2, 6)$.

7. $a = 5\text{ km}$, $d = 2\text{ km}$.

From formula book, $u_n = a + (n-1)d$, $S_n = \frac{n}{2}(2a + (n-1)d)$.

(a) $u_4 = a + (n-1)d = 5 + (4-1)2 = 5 + 6 = 11\text{ km}$.

(b) $u_n = a + (n-1)d = 5 + (n-1)2 = 5 + 2n - 2 = 2n + 3\text{ km}$

(c) Total $= S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{\cancel{2}}(\cancel{2}\times 5 + (n-1)\cancel{2})$

$= n(5 + n - 1) = n(n+4)\text{ km}$

(d) from (b), $2n + 3 = 43$, $2n = 40$, $n = 20^{\text{th}}$ Saturday.

(e) Using $n$ value from (d):

Total $= S_n = n(n+4) = 20 \times 24 = 480\text{ km}$

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8. (a) If a quadratic $ax^2 + bx + c = 0$ has no real roots,

$b^2 - 4ac < 0$

Think: if you tried to solve it using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, you would be trying to do $\sqrt{\text{negative number}}$

For $2qx^2 + qx - 1 = 0$,

$a = 2q$, $b = q$, $c = -1$

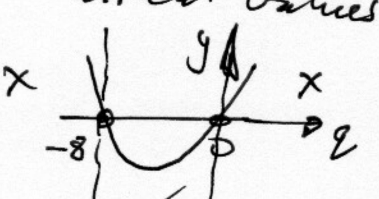
$\therefore q^2 - 4(2q)(-1) < 0$

$q^2 + 8q < 0$.

(b) Remember you are finding $q$, not solving the equation for $x$.

Factorise: $q(q+8) < 0$. Quadratic inequality

Critical values, $q(q+8) = 0$ at $q = -8$ or $0$



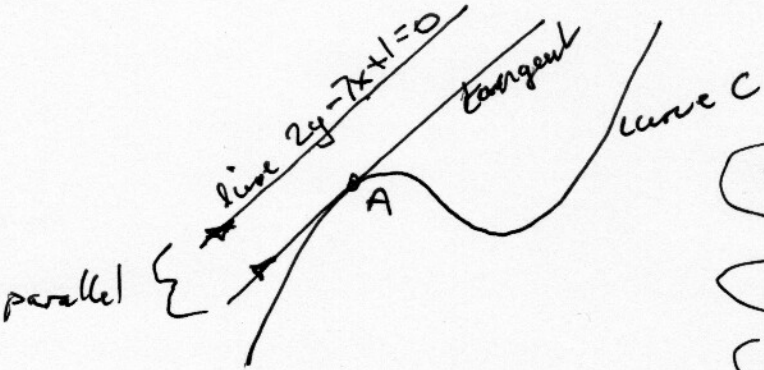

If $y = q(q+8)$, need $y < 0$

$\therefore -8 < q < 0$

9 (a) $y = kx^3 - x^2 + x - 5$

$dy/dx = 3kx^2 - 2x + 1$

(b) Read the question carefully and think what it means. A sketch will help.




Find gradient of line

= gradient of tangent

= dy/dx → contains x and k

We know x, so we have an equation in k

$2y - 7x + 1 = 0$

$\therefore 2y = 7x - 1,\ y = \frac{7}{2}x - \frac{1}{2} \Rightarrow$ gradient is $\frac{7}{2}$

At $x = -\frac{1}{2}$, $dy/dx = 3k\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1 = \frac{3}{4}k + 2 = 3\frac{1}{2}$

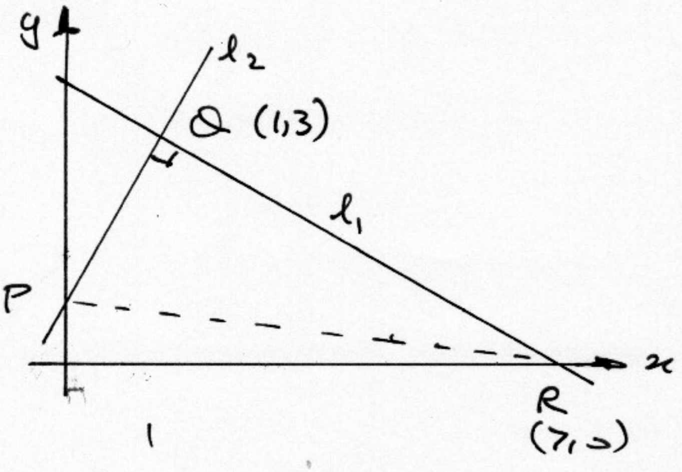
$\therefore \frac{3}{4}k = 1\frac{1}{2},\ 3k = 6,\ \underline{k = 2}$.

(c) Knowing $k = 2$ now, ~~dy/dx~~

$y = 2x^3 - x^2 + x - 5.$

At $x = -\frac{1}{2}$, $y = 2\left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right) + \left(-\frac{1}{2}\right) - 5 = -6$.

10.




(a). $QR^2 = (x_2-x_1)^2+(y_2-y_1)^2 = (7-1)^2+(0-3)^2 = 6^2+3^2 = 36+9 = 45.$

$QR = \sqrt{45} = \sqrt{9\times5} = 3\sqrt{5} \quad = a\sqrt{5}, \quad a=3.$

(b) Gradient $m_1$ of line $l_1$ is $m_1 = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{0-3}{7-1} = \dfrac{-3}{6} = -\dfrac{1}{2}$

[check, slopes down, is negative ✓].

$l_2 \perp$ to $l_1$ so $m_2 = \dfrac{-1}{m_1} = \dfrac{-1}{(-1/2)} = 2$

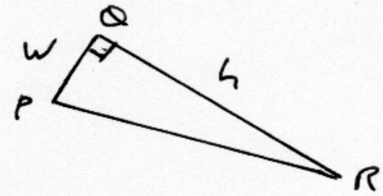
$y-y_1 = m(x-x_1), \quad y-3 = 2(x-1) = 2x-2$

$\therefore y = 2x+1$

(c) On the y-axis, $x=0$, $y = 2(0)+1 = 1$
P is at $(0,1)$.

(d)

Must spot that $l_2$ is perpendicular to $l_1$ (read the question) so we can determine width and height:




$QR = 3\sqrt{5}$ from (a)

$QP = \sqrt{(1-0)^2+(3-1)^2} = \sqrt{1+4} = \sqrt{5}.$

Area $= \frac{1}{2}wh = \frac{1}{2}\sqrt{5}(3\sqrt{5})$

$= \frac{15}{2} = 7\frac{1}{2}.$

11. (a) $\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2} = \frac{x^4+6x^2+9}{x^2} = x^2+6+9x^{-2}$.

[the $x \neq 0$ tells you not to worry about it being infinite at $x=0$ $\left(9x^{-2} = \frac{9}{0} = \infty\right]$ - only use it for other $x$ values]. 

(b) $y = \int dy/dx \, dx = \int x^2+6+9x^{-2} \, dx$

$= \frac{1}{3}x^3+6x-9x^{-1}+C$

<u>Now find C</u> :

At $x=3$, $y = 1/3(3^3)+6\times3-\frac{9}{3}+C$

$= 9+18-3+C = 24+C$

$= 20$, since $(3,20)$ is on the curve.

$\therefore C = -4$

$\underline{y = 1/3x^3+6x-9x^{-1}-4}$

← Don't forget the final answer, there's 1 mark for it!