# C1 MAY 2007

1. $(3+\sqrt{5})(3-\sqrt{5}) = 9 + 3\sqrt{5} - 3\sqrt{5} - (\sqrt{5})^2$
$= 9 - 5 = 4$

2(a) $8^{4/3} = (\sqrt[3]{8})^4 = 2^4 = 16$

(b) $\frac{15x^{4/3}}{3x} = \left(\frac{15}{3}\right) x^{4/3 - 1} = 5x^{1/3}$

3. $y = 3x^2 + 4x^{1/2}$

a) $dy/dx = 3(2x) + 4(\tfrac{1}{2}x^{-1/2}) = 6x + 2x^{-1/2}$

b) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = 6 + 2(-\tfrac{1}{2}x^{-3/2}) = 6 - x^{-3/2}$

(c) $\int y\,dx = \int 3x^2 + 4x^{1/2}\,dx = 3\left(\frac{x^3}{3}\right) + 4\left(\frac{x^{3/2}}{3/2}\right) + c$

$= x^3 + \frac{8}{3}x^{3/2} + c$

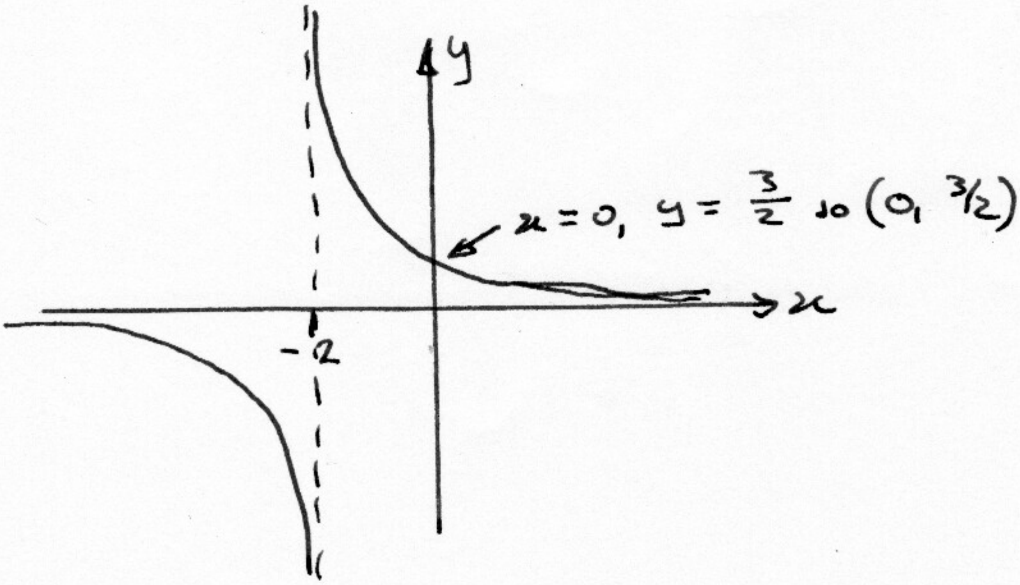
4. $a = 5,\ d = 2$ for series $5 + 7 + 9 + \ldots$

a) $u_n = a + (n-1)d$, $u_{200} = 5 + 199 \times 2 = 403p$

b) $S_n = \frac{n}{2}(a + l)$, $S_{200} = \frac{200}{2}(5 + 403)$

$= 408 \times 100p = £408$

5. (a) $f(x) = 3/x$, $f(x+2) = \frac{3}{x+2}$, shift $\xleftarrow{2}$ to left




(b) Asymptotes are $y=0$ ($x$-axis)

and $x=-2$.

6. (a) $y = x-4$

$2x^2 - xy = 8$

$\therefore\ 2x^2 - x(x-4) = 2x^2 - x^2 + 4x$

$= x^2 + 4x = 8.$

$x^2 + 4x - 8 = 0$

(b) Solving $x^2+4x-8=0$,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{16+32}}{2} = -2 \pm \frac{\sqrt{48}}{2}$$

$$= -2 \pm \frac{\sqrt{16\times 3}}{2} = -2 \pm \frac{4\sqrt{3}}{2} = -2 \pm 2\sqrt{3}$$

Then $y = x-4$

So at $x = -2+2\sqrt{3}$, $y = -6+2\sqrt{3}$

at $x = -2-2\sqrt{3}$, $y = -6-2\sqrt{3}$.

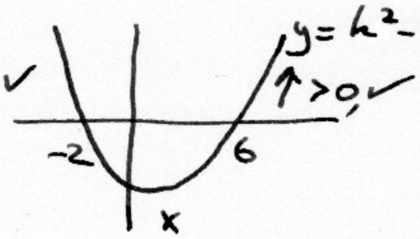
7. Different real roots $\Rightarrow$ discriminant $b^2-4ac>0$

(a) $b^2-4ac = k^2-4(k+3) = k^2-4k-12$

$\therefore\ k^2-4k-12>0$

(b) $k^2-4k-12 = (k-6)(k+2) = 0$ for critical values,

$k=-2$ or $6$.




$\therefore\ k<-2$ or $k>6$

8. $a_1 = k, \quad a_{n+1} = 3a_n+5$

(a) $a_2 = 3k+5$

(b) $a_3 = 3(3k+5)+5 = 9k+15+5 = 9k+20$

(c) (i) $a_4 = 3(9k+20)+5 = 27k+65$

$$\sum_{r=1}^{4} a_r = a_1+a_2+a_3+a_4$$

$$= k+(3k+5)+(9k+20)+(27k+65)$$

$$= 40k+90,\ \text{so}$$

(ii) $\sum_{r=1}^{4} a_r = 10(4k+9)$ so is divisible by 10.

9. (a) Think: a differential equation question, integrate to convert $f'(x)$ into $f(x)$, use the boundary condition to set the value of the constant $c$

$$f(x) = \int f'(x)\,dx = \int 6x^2 - 10x - 12\,dx$$

$$= 6\left(\frac{x^3}{3}\right) - 10\left(\frac{x^2}{2}\right) - 12x + c$$

$$= 2x^3 - 5x^2 - 12x + c$$

At $x = 5$,

$$y = f(x) = 2(5^3) - 5(5^2) - 12 \times 5 + c$$

$$= 2 \times 125 - 5 \times 25 - 60 + c$$

$$= 250 - 185 + c = 65 + c = 65$$

at $(5, 65)$.

$\therefore c = 0$,

$$f(x) = 2x^3 - 5x^2 - 12x$$

(b) $f(x) = x(2x^2 - 5x - 12)$

Factors of $2x^2 - 5x - 12$, $\quad 2 \times -12 = -24$,

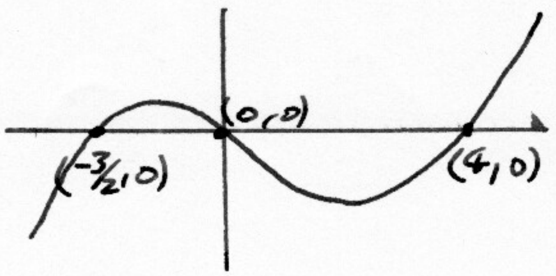
factors $-8 \times 3$.

$$2x^2 - 8x + 3x - 12 = 2x(x-4) + 3(x-4)$$

$$= (2x+3)(x-4)$$

$$\therefore f(x) = x(2x+3)(x-4)$$

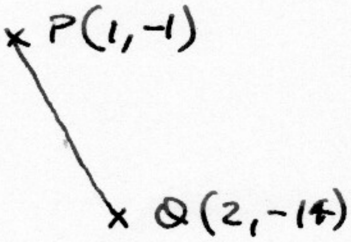
(c) Cuts x-axis ($y=0$) at $x = 0,\ 4,\ -3/2$. Cubic, $a > 0$ $\nearrow$.



10. $y = x^2(x-6) + \frac{4}{x}$

(a) P: $x=1$, $y = 1(1-6) + \frac{4}{1} = -5+4 = -1$

Q: $x=2$, $y = 4(2-6) + \frac{4}{2} = -16+2 = -14$




$$PQ = \sqrt{(2-1)^2 + (-14-(-1))^2} = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$$

(b) Parallel $\Rightarrow$ must show gradients are the same.

$\cancel{dy/dx}$ $y = x^3 - 6x^2 + 4x^{-1}$

$dy/dx = 3x^2 - 12x + 4(-x^{-2}) = 3x^2 - 12x - 4x^{-2}$.

At $x=1$, $dy/dx = 3 - 12 - \frac{4}{1^2} = -13$

At $x=2$, $dy/dx = 3(2^2) - 12(2) - \frac{4}{2^2} = 12 - 24 - 1$
$= -13$.

Gradients of curves equal at P & Q $\Rightarrow$ tangents are parallel.

(c) Gradient of normal $= \frac{-1}{-13} = 1/13$.

Using $y - y_1 = m(x - x_1)$,

$y - (-1) = \frac{1}{13}(x-1)$

(×13) $13y + 13 = x - 1$, $\quad x - 13y - 14 = 0$

11. (a) $l_2$ is $3x + 2y - 8 = 0$

$$2y = -3x + 8, \quad y = -\tfrac{3}{2}x + 4$$

gradient $-3/2$

(b) At intersection,

$$y = 3x + 2 = (-3/2)x + 4$$

$$4\tfrac{1}{2}x = 4 - 2 = 2, \quad x = (2/9) \times 2 = 4/9$$

$$y = 3x + 2 = \frac{4}{3} + 2 = 3\tfrac{1}{3}$$

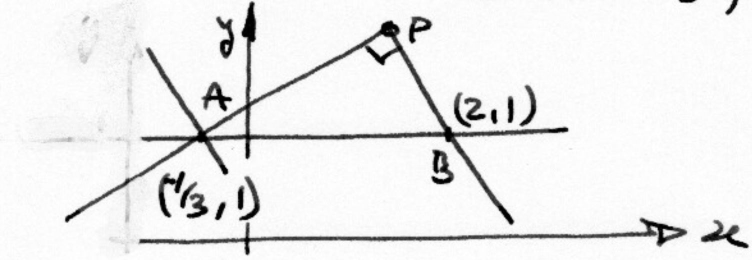
$\therefore$ P is $(4/9, 3\tfrac{1}{3})$

(c) When $l_1$ crosses $y = 1$,

$$y = 3x + 2 = 1, \quad 3x = -1, \quad \underline{x = -1/3}$$

When $l_2$ crosses $y = 1$,

$$y = -3/2x + 4 = 1, \quad -3/2x = -3, \quad 1/2x = 1, \quad \underline{x = 2}$$




Area of $ABP = \tfrac{1}{2}bh = \tfrac{1}{2}\left(2 + \tfrac{1}{3}\right)\left(3\tfrac{1}{3} - 1\right)$

$$= \tfrac{1}{2} \times 7/3 \times \frac{7}{3} = \frac{49}{18}.$$