C1 JAN 2011

1(a) $16^{-1/4} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$

(b) $x\left(2x^{-1/4}\right)^4 = x\left(2^4\right)x^{(-1/4\times 4)} = 16\times x^{-1} = 16$

2. $\int 12x^5 - 3x^2 + 4x^{1/3}\,dx = 12\frac{x^6}{6} - 3\left(\frac{x^3}{3}\right) + 4\frac{x^{4/3}}{4/3} + c$

$= 2x^6 - x^3 + 3x^{4/3} + c$

3. $\frac{5-2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{5\sqrt{3}+5-6-2\sqrt{3}}{3-1} = \frac{3\sqrt{3}-1}{2} = -\frac{1}{2}+\frac{3}{2}\sqrt{3}$

4. $a_1 = 2, \quad a_{n+1} = 3a_n - c$

(a) $a_2 = 3a_1 - c = 3\times 2 - c = 6 - c$

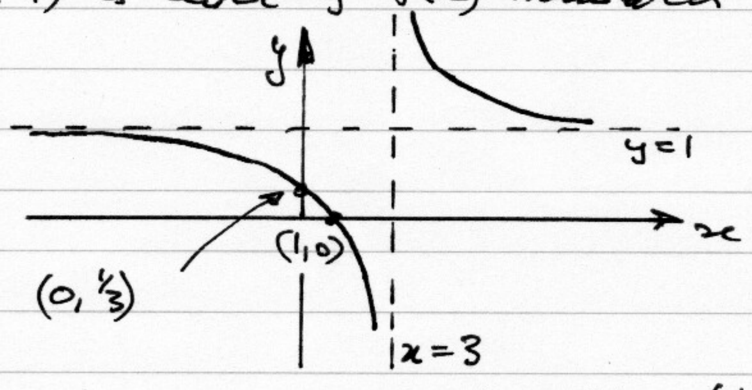
(b) $\sum_{i=1}^{3} a_i = a_1 + a_2 + a_3 = 0$

$a_3 = 3a_2 - c = 3(6-c) - c = 18 - 3c - c = 18 - 4c$

$\therefore a_1 + a_2 + a_3 = 2 + 6 - c + 18 - 4c = 26 - 5c = 0$

$c = \frac{26}{5} = 5.2$

5(a) $y = f(x-1)$ is curve $y = f(x)$ translated 1 to right →.



(b) Intersects x-axis at (1,0) (translation of origin 1 to right).

On y-axis $x = 0$, $x-1 = -1$, $y = \frac{-1}{-1-2} = 1/3$

6 (a) $S_n = \frac{n}{2}(2a + (n-1)d)$, $S_{10} = 162$

$\therefore S_{10} = \frac{10}{2}(2a + 9d) = 10a + 45d = 162$

(b) n^{th} term $u_n = a + (n-1)d$ $\therefore$ 6^{th} term is $a + 5d$

$a + 5d = 17$

(c) Eliminate d: $9(a+5d) = 9a + 45d = 9 \times 17 = 153$

$(10a + 45d) - (9a + 45d) = 162 - 153$

$\therefore a = 9$

$9 + 5d = 17$, $5d = 8$, $d = \frac{8}{5} = 1.6$

7. $f'(x) = 12x^2 - 8x + 1$

$f(x) = \int f'(x)\,dx = 12\left(\frac{x^3}{3}\right) - 8\left(\frac{x^2}{2}\right) + x + c$

$= 4x^3 - 4x^2 + x + c$

At $x = -1$, $y = f(x) = 0$

$\therefore 4(-1)^3 - 4(-1)^2 - 1 + c = 0$

$-4 - 4 - 1 + c = -9 + c = 0$, $c = 9$

$\therefore f(x) = 4x^3 - 4x^2 + x + 9$

8 (a) Let $x^2 + (k-3)x + (3-2k) = ax^2 + bx + c = 0$,

$a = 1$, $b = k-3$, $c = 3-2k$

For two distinct roots the discriminant $b^2 - 4ac > 0$

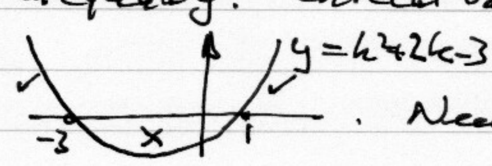
$\therefore (k-3)^2 - 4(1)(3-2k) > 0$

$k^2 - 6k + 9 - 12 + 8k > 0$

$k^2 + 2k - 3 > 0$

(b) Quadratic inequality. Critical values $(k+3)(k-1) = 0$,

$k = -3$ or 1.



Need > 0 $\therefore$ $k < -3$, $k > 1$

9. (a) $2y - 3x - k = 0$.

$(1,4)$ is on the line $\therefore 2\times 4 - 3\times 1 - k = 0$, $8 - 3 - k = 0$,
$5 - k = 0$, $k = 5$.

(b) $2y = 3x + 5$, $y = \frac{3}{2}x + \frac{5}{2}$, gradient $\frac{3}{2}$.

(c) Perpendicular to L_1, need gradient $\frac{-1}{(3/2)} = -\frac{2}{3}$

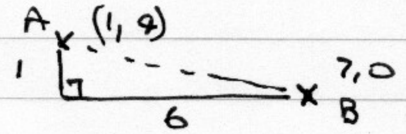
$y - y_1 = m(x - x_1) \Rightarrow y - 4 = -\frac{2}{3}(x - 1)$

$\times 3$: $3y - 12 = -2(x-1) = -2x + 2$

$\therefore 2x + 3y - 14 = 0$

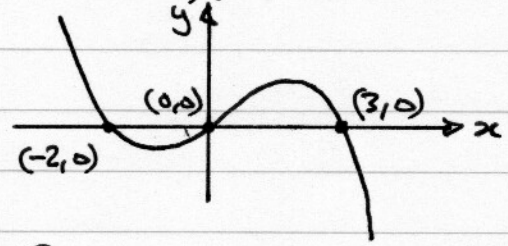
(d) On x-axis, $y = 0$

$\therefore 2x - 14 = 0$, $x = 7$, B is $(7,0)$.

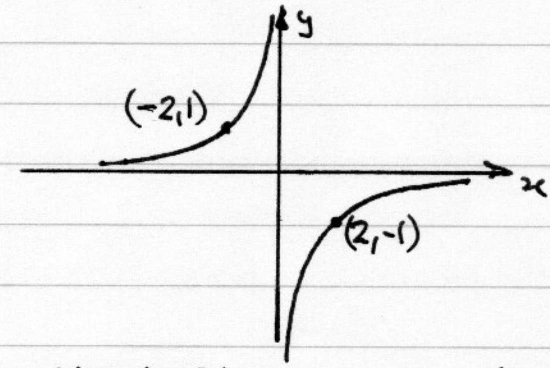
(e) 

Length $AB = \sqrt{1^2 + 6^2} = \sqrt{37}$

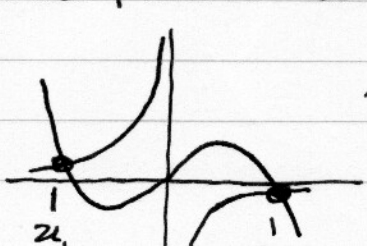
10 (a)(i) $y = x(x+2)(3-x)$ is a $-x^3$ cubic with roots $0, -2, 3$



(ii) $y = -\frac{2}{x}$, reciprocal curve reflected in x-axis:



(b) $x(x+2)(3-x) + \frac{2}{x} = 0 \Rightarrow x(x+2)(3-x) = -\frac{2}{x}$, solutions are x-values of the intersection points of the two curves.



Two roots!

11.(a) $y = \frac{1}{2}x^3 - 9x^{3/2} + 8x^{-1} + 30$

$$\frac{dy}{dx} = \tfrac{1}{2}(3x^2) - 9\left(\tfrac{3}{2}x^{1/2}\right) + 8\left(-x^{-2}\right) + 0$$
$$= \frac{3}{2}x^2 - \frac{27}{2}x^{1/2} - 8x^{-2}$$

(b) At $x = 4$, we expect $y = \frac{1}{2}(4^3) - 9(4^{3/2}) + \frac{8}{4} + 30$
$$= 32 - 9 \times 8 + 2 + 30$$
$$= 32 - 72 + 32 = -8$$
$\therefore$ $(4, -8)$ is on the curve.

(c) At $x = 4$, $dy/dx = \tfrac{3}{2}(4^2) - \frac{27}{2}(2) - \frac{8}{16}$
$$= 24 - 27 - \tfrac{1}{2} = -3\tfrac{1}{2} = -\tfrac{7}{2}$$
$\therefore\ m_N = \frac{-1}{m_T} = \frac{-1}{(-7/2)} = 2/7$

$y - y_1 = m(x - x_1)$, $\quad y - (-8) = \tfrac{2}{7}(x - 4)$
$= y + 8$

$(\times 7)$: $7y + 56 = 2(x-4) = 2x - 8$

$\therefore$ $2x - 7y - 64 = 0$