JANUARY 2010 C1

1. $y = x^4 + x^{1/3} + 3, \quad dy/dx = 4x^3 + \frac{1}{3}x^{-2/3}$

2. (a) $(7+\sqrt{5})(3-\sqrt{5}) = 21 - 7\sqrt{5} + 3\sqrt{5} - 5$

$= 16 - 4\sqrt{5}$

(b) $\frac{7+\sqrt{5}}{3+\sqrt{5}} = \frac{(7+\sqrt{5})}{(3+\sqrt{5})}\left(\frac{3-\sqrt{5}}{3-\sqrt{5}}\right) = \frac{16-4\sqrt{5}}{9-5} = 4-\sqrt{5}$

3. l_1 is line $3x + 5y - 2 = 0$

(a) $5y = -3x + 2, \quad y = \frac{-3}{5}x + \frac{2}{5},$

$\therefore$ gradient $= -3/5$.

(b) gradient of $l_2 = \frac{-1}{(-3/5)} = 5/3$

$y - y_1 = m(x - x_1), \quad y - 1 = 5/3(x-3) = 5/3x - 5$

$\therefore y = 5/3\, x - 4$

4. $dy/dx = 5x^{-1/2} + x\sqrt{x} = 5x^{-1/2} + x^{1\frac{1}{2}}$

$y = \int dy/dx \, dx = \int 5x^{-1/2} + x^{3/2} dx$

$= 5\left(\frac{x^{1/2}}{(1/2)}\right) + \frac{x^{5/2}}{(5/2)} + c$

$= 10x^{1/2} + 2/5\, x^{5/2} + c.$

At $x = 4$, $y = 10\sqrt{4} + 2/5(4^2\sqrt{4}) + c = 20 + \frac{64}{5} + c$

$= 20 + \frac{128}{10} + c = 32 \cdot 8 + c = 35.$

$\therefore c = 35 - 32 \cdot 8 = 2 \cdot 2$

$y = 10x^{1/2} + 2/5\, x^{5/2} + 2\frac{1}{5}$

5. $y - 3x + 2 = 0 \longrightarrow y = 3x - 2$

$y^2 - x - 6x^2 = 0 \longrightarrow (3x-2)^2 - x - 6x^2 = 0$

$\therefore (9x^2 - 12x + 4) - x - 6x^2$

$= 3x^2 - 13x + 4 = 0$

$ac = 12 = -1 \times -12$ $\qquad (3x - \frac{1}{1})(1x - \frac{12}{3})$

$= (3x-1)(x-4) = 0$

$\therefore x = 1/3$ or 4.

$x = 1/3 \Rightarrow y = 3x - 2 = 1 - 2 = -1 \qquad \underline{\underline{(1/3, -1)}}$

$x = 4 \Rightarrow \underline{y = 12 - 2 = 10} \qquad \underline{\underline{(4, 10)}}$

6.(a) Must write y as a series of x^n terms <u>before differentiating</u>.

$y = \dfrac{(x+3)(x-8)}{x} = \dfrac{x^2 + 3x - 8x - 24}{x} = x - 5 - 24x^{-1}$

$\therefore dy/dx = 1 - 24(-x^{-2}) = 1 + 24x^{-2}$

(b) At $x = 2$, $y = \dfrac{5(-6)}{2} = -15$

$dy/dx = 1 + \dfrac{24}{2^2} = 1 + 6 = 7$

$\therefore y - (-15) = 7(x-2) = 7x - 14$

$= y + 15$.

$\underline{y = 7x - 29}$

7. <u>Arithmetic sequence</u>. $a = £150$, $d = £10$

(a) $u_n = a + (n-1)d$, $u_{10} = a + 9d = 150 + 90 = £240$

(b) $S_n = n/2(2a + (n-1)d)$,

$S_{20} = \frac{20}{2}(300 + 19 \times 10) = 10 \times 490 = £4900$

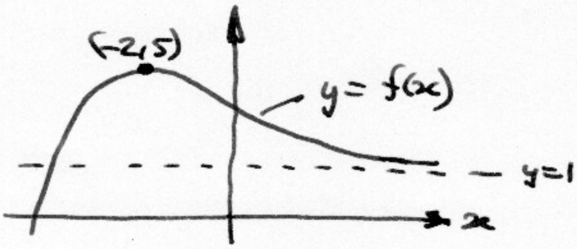
(c) For Kevin, $a = A$, $d = 30$.

His $S_{20} = \frac{20}{2}(2A + 19 \times 30) = 2 \times 4900$

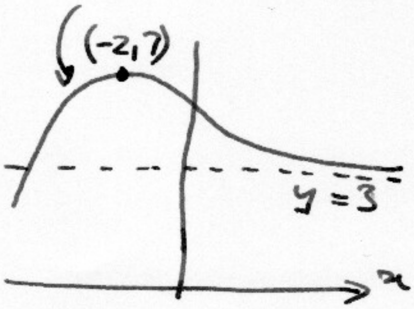
$\therefore 2A + 570 = 2 \times 490 = 980$

$2A = 410$, $\underline{A = £205}$

8.

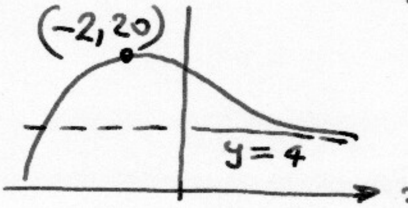


(a) $y = f(x)+2$, moves ↑ 2 { Operation outside $f(x)$, ∴ affects y. Adding = translation. Changing y ∴ moved in direction you expect! }



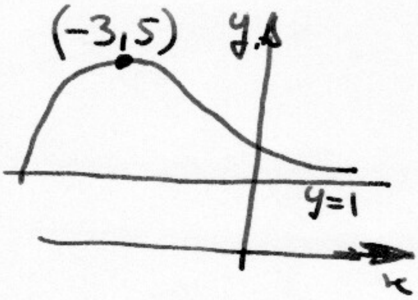
↑ +2

(b) $y = 4f(x)$ { Outside $f(x)$, affects y. Multiplication = stretch. y ∴ in expected direction, 4 × bigger }.



↕ ×4

(c) $y = f(x+1)$ { inside (x) brackets ∴ need a different $\underline{x}$ to get same $\underline{y}$. Addition = translation. x gets 1 added to it by the "$x+1$" so start with a <u>smaller value</u> ∴ moves $\xleftarrow{1}$ to left }.



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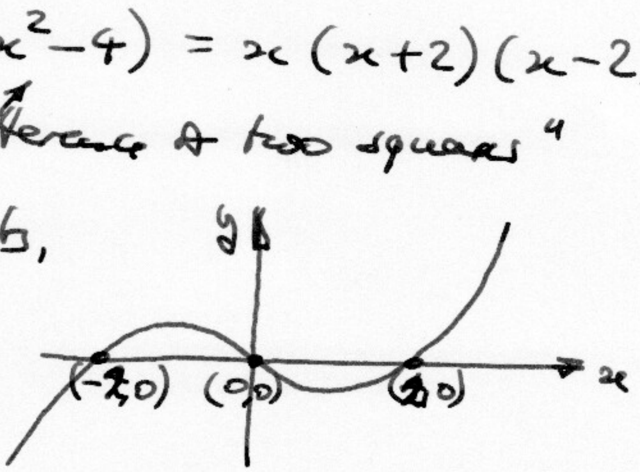
9. (a) $x^3-4x = x(x^2-4) = x(x+2)(x-2)$

"difference of two squares"

(b) Cubic, $+x^3$, 3 roots,



9(c) At $x=-1$, $y=(-1)^3-4(-1)=3$ (A)

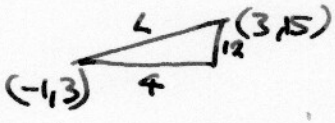
At $x=3$, $y=3^2-4\times3=15$ (B)

gradient $\dfrac{15-3}{3-(-1)} = \dfrac{12}{4} = 3$.

$y-3 = 3(x-(-1)) = 3(x+1) = 3x+3$

$\therefore y = 3x+6$.

(d)



$L^2 = 4^2+12^2 = 16+144 = 160$

$\therefore L = \sqrt{16\times10} = 4\sqrt{10}$

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10. (a) $f(x) = x^2 + 4kx + (3+11k)$

$= (x+2k)^2 - (2k)^2 + 3 + 11k$

$= \underbrace{(x+2k)^2}_{p} + \underbrace{(3+11k-4k^2)}_{q}$

(b)

If $f(x)=0$ has no real roots,

since $(x+2k)^2 = 4k^2-11k-3$,

$4k^2-11k-3<0$.

{ or use $b^2-4ac<0$, $\therefore (4k)^2-4(3+11k)<0$

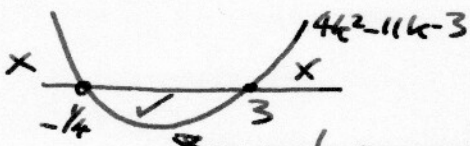
$\div 4$, $4k^2-11k-3<0$. }.

Quadratic inequality.

Factorise, $ac=-12=-12\times1$, $(4k+\frac{1}{1})(1k-\frac{12}{4}) = (4k+1)(k-3)$

$\therefore$ "Critical values" are $-1/4, 3$.

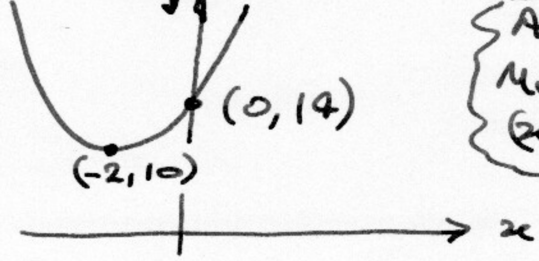
sketch:



want <0 region, $\boxed{-1/4<k<3}$

(c) $k=1$ (check, this is between $-1/4$ and $+3$ ✓).

$\therefore f(x) = (x+2)^2 + (3+11-4) = (x+2)^2+10$



{ At $x=0$, $y=2^2+10=14$.
Minimum $y=10$, where $(x+2)^2=0$ i.e. $x=-2$. }