C1 JAN 2009

1.a) $125^{1/3} = \sqrt[3]{125} = 5$

b) $125^{-2/3} = \left(125^{1/3}\right)^{-2} = 5^{-2} = \frac{1}{25}$

2. $\int 12x^5 - 8x^3 + 3\,dx = 12\left(\frac{x^6}{6}\right) - 8\left(\frac{x^4}{4}\right) + 3x + c = 2x^6 - 2x^4 + 3x + c$

3. $(\sqrt{7}+2)(\sqrt{7}-2) = (\sqrt{7})^2 - 2\sqrt{7} + 2\sqrt{7} - 2^2 = 7 - 4 = 3$

4. $f'(x) = 3x^2 - 3x^{1/2} - 7 \quad \therefore f(x) = \int f'(x)\,dx = 3\left(\frac{x^3}{3}\right) - 3\frac{x^{3/2}}{(3/2)} - 7x + c$

$= x^3 - 2x^{3/2} - 7x + c$

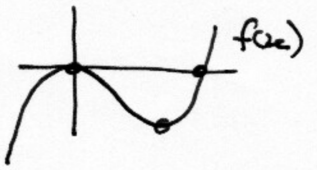
At $x = 4$, $y = 22$

$\therefore 4^3 - 2(4\sqrt{4}) - 28 + c = 22$

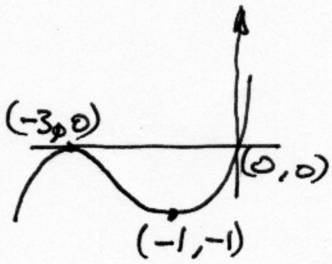
$64 - 16 - 28 + c = 22$

$= 20 + c$, $c = 2$, $f(x) = x^3 - 2x^{3/2} - 7x + 2$

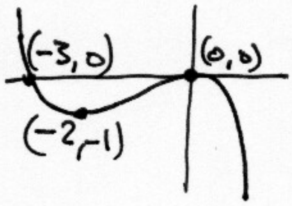
5.



(a) $f(x+3)$ moves $\xleftarrow{3}$



(b) $y = f(-x)$ reflects in y-axis



6.
(a) $\frac{2x^2 - x^{3/2}}{x^{1/2}} = 2x^{1\frac{1}{2}} - x^1 \quad \therefore p = 1\frac{1}{2}, \quad q = 1$

Laws of indices $\frac{x^2}{x^{1/2}} = x^{2-\frac{1}{2}} = x^{1\frac{1}{2}}$

(b) $y = 5x^4 - 3 + 2x^{3/2} - x$

$dy/dx = 20x^3 + 3x^{1/2} - 1$

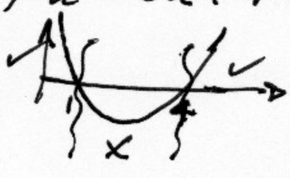
7. 2 real roots $\therefore b^2 - 4ac > 0$

$kx^2 + 4x + (5-k) = 0$, $a = k$, $b = 4$, $c = 5 - k$

$\therefore 4^2 - 4k(5-k) > 0$

$4k^2 - 20k + 16 > 0$, $(\div 4)$, $k^2 - 5k + 4 > 0$

Critical values: $(k-4)(k-1) = 0$,
$k = 1$ or 4

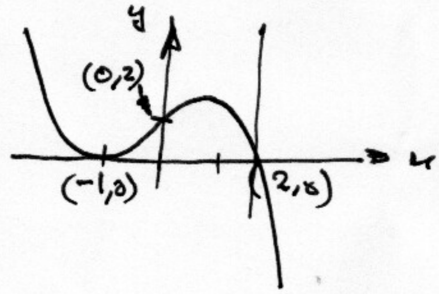


$k < 1$ or $k > 4$

8 (a) $y = (x+1)^2(2-x)$. At $x=1$, $y=a$

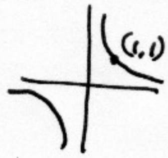
$\therefore a = 2^2(2-1) = 4$

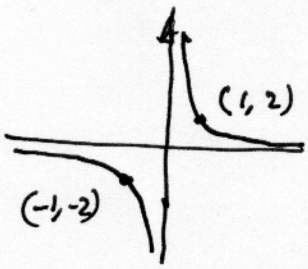
(b)(i) $-x^3$ cubic, roots -1 (repeated) and 2.



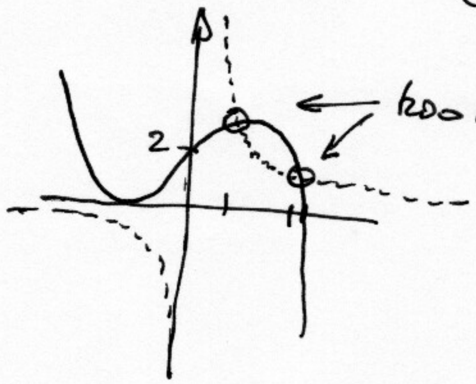
At $x=0$, $y = 1^2(2) = 2$

(ii) $y = 1/x$ so $y = 2/x$ is stretched $\updownarrow \times 2$





(c)



two real solutions (x-values here)

9. (a) $u_{18} = a + 17d = 25$ (from $u_n = a + (n-1)d$)

$u_{21} = a + 20d = 32\frac{1}{2}$

(b) $\therefore 3d = 7\frac{1}{2}$, $d = 2\frac{1}{2}$

$a = 32\frac{1}{2} - 20d = 32\frac{1}{2} - 50 = -17\frac{1}{2}$

(c) $S_n = 2750 = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(-35 + 2\frac{1}{2}(n-1))$

(×2) $5500 = n(2\frac{1}{2}n - 37\frac{1}{2})$, $11000 = n(5n - 75) = 5n^2 - 75n$

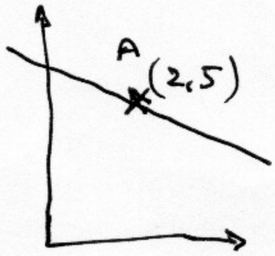
(÷5) $n^2 - 15n = 2200 = 55 \times 40$

(d) $n^2 - 15n - 2200 = 0$

$(n-55)(n+40) = 0$, $n = 55$ or -40

but need $n > 0$ $\therefore$ $\underline{n = 55}$.

10. (a)



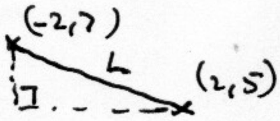
$y - 5 = -\tfrac{1}{2}(x-2) = -\tfrac{1}{2}x + 1$

$\therefore y = -\tfrac{1}{2}x + 6$

(b) On L, at $x = -2$, expect $y = -\tfrac{1}{2}(-2) + 6 = 1 + 6 = 7$.

$\therefore (-2,7)$ is on L.

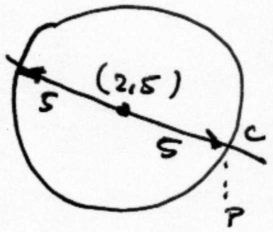
(c)



$L^2 = (2-(-2))^2 + (7-5)^2 = 4^2 + 2^2 = 20$

$\therefore$ length $AB = \sqrt{20} = 2\sqrt{5}$.

(d)



Circle centre $(2,5)$ radius 5,

$(x-2)^2 + (y-5)^2 = 5^2$

intersects $y = -\tfrac{1}{2}x + 6$

$\therefore (x-2)^2 + (-\tfrac{1}{2}x + 6 - 5)^2 = 25$

$x = p \Rightarrow (p-2)^2 + (1 - \tfrac{1}{2}p)^2 = p^2 - 4p + 4 + 1 - p + \tfrac{1}{4}p^2 = 25$

$\tfrac{5}{4}p^2 - 5p + 5 = 25$, $\boxed{\times \tfrac{4}{5}}$ $p^2 - 4p + 4 = 20$,

$p^2 - 4p - 16 = 0$.

11. $y = 9 - 4x - 8/x$

(a) At $x = 2$, $y = 9 - 8 - 4 = -3$

$dy/dx = -4 - 8(-x^{-2}) = -4 + 8/x^2$

$m_T = -4 + 8/4 = -2$,

$y - (-3) = -2(x-2) = -2x + 4$

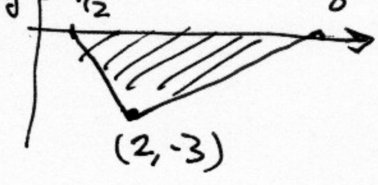
$y = -2x + 1$, $y = 1 - 2x$ ✓.

(b) $m_N = \frac{-1}{-2} = \tfrac{1}{2}$. $y - (-3) = \tfrac{1}{2}(x-2) = \tfrac{1}{2}x - 1$

$y = \tfrac{1}{2}x - 4$

(c) Tangent $y = 1 - 2x$, on x-axis $y = 0$ $\therefore 1 - 2x = 0$, $x = \tfrac{1}{2}$

Normal $y = \tfrac{1}{2}x - 4$, .. $y = 0$, $\tfrac{1}{2}x = 4$, $x = 8$



area $= \tfrac{1}{2}wh = \tfrac{1}{2}(8 - \tfrac{1}{2})3$

$= \tfrac{1}{2}(\tfrac{15}{2})3 = \tfrac{45}{4} = 11\tfrac{1}{4}$