C1 January 2008

1. $\int 3x^2 + 4x^5 - 7\,dx = 3\left(\frac{x^3}{3}\right) + 4\left(\frac{x^6}{6}\right) - 7x + c$

$= x^3 + \frac{2}{3}x^6 - 7x + c$

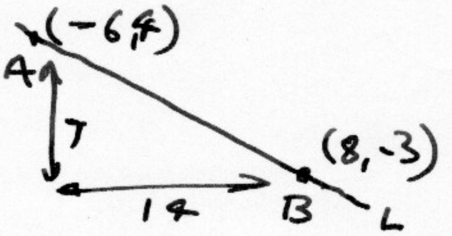
2(a) $16^{1/4} = \sqrt[4]{16} = 2$

(b) $(16x^{12})^{3/4} = 16^{3/4}(x^{12})^{3/4} = 2^3\, x^{12 \times 3/4} = 8x^9$

{ Hint: $(ab)^n = a^n b^n$ and $(x^p)^q = x^{pq}$ }

3. $\left(\frac{5-\sqrt{3}}{2+\sqrt{3}}\right)\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = \frac{10-5\sqrt{3}-2\sqrt{3}+3}{4+2\sqrt{3}-2\sqrt{3}-3} = 13-7\sqrt{3}$

4. (a)



gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-4}{8-(-6)}$

$= \frac{-7}{14} = -\frac{1}{2}$

$y - 4 = -\frac{1}{2}(x - (-6))$

$= -\frac{1}{2}(x + 6)$

$2y - 8 = -x - 6, \quad x + 2y - 2 = 0$

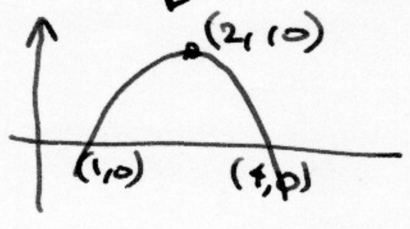
(b) $\sqrt{14^2 + 7^2} = \sqrt{7^2}\sqrt{2^2+1} = 7\sqrt{5}$

5.(a) $\frac{2x^{1/2}}{x^1} + \frac{3}{x^1} = 2x^{-1/2} + 3x^{-1}$

(b) $y = 5x - 7 + 2x^{-1/2} + 3x^{-1}$

$dy/dx = 5 - x^{-3/2} - 3x^{-2}$

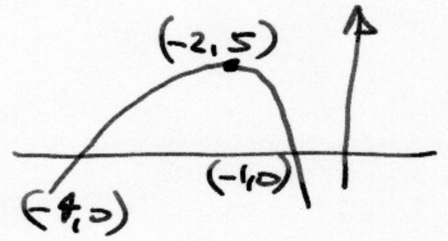
6.(a) stretch $\updownarrow \times 2$ for $y = 2f(x)$



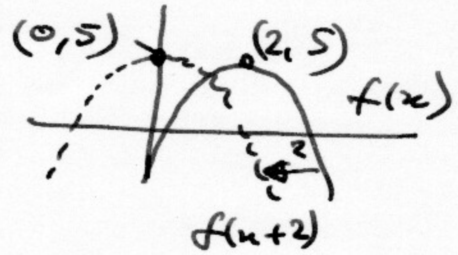
6(b) $y = f(-x)$ is $y = f(x)$ reflected $\leftrightarrow$ in y-axis

eg you know $f(2) = 5$ from question.

If $x = -2$, $f(-x) = f(2) = 5 \Rightarrow (-2, 5)$.



(c) Shifts $\xleftarrow{2}$ so need $f(x+2)$, $a = 2$.



7(a) $x_1 = 1$, $x_2 = x_1(p + x_1) = 1(p+1) = p+1$

(b) $x_3 = x_2(p + x_2) = (p+1)(p + p + 1)$

$= (p+1)(2p+1) = 2p^2 + 3p + 1$

(c) $x_3 = 1$ (equation)

use $x_3 = 2p^2 + 3p + 1$ definition

$\therefore 2p^2 + 3p + 1 = 1$, $\quad 2p^2 + 3p = 0$

$= p(2p+3)$

$\therefore p = 0$ or $-3/2$

but $p \neq 0$ $\therefore p = -3/2$

(d) Terms oscillate $1, p+1, 1, p+1, 1, p+1 \ldots\ldots$

repeats!

$\therefore x_{2008} = p + 1 = -1/2$

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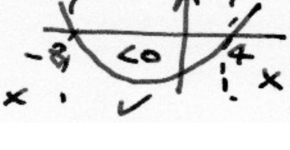
8. Quadratic, $x^2 + kx + 8 = k$ $\quad \therefore x^2 + \overset{b}{k}x + \overset{c}{(8-k)} = 0$

(a) No real roots $\therefore b^2 - 4ac < 0$

$k^2 - 4(8-k) < 0$, $\quad k^2 + 4k - 32 < 0$

(b) Quad. inequality. Critical values $(k+8)(k-4) = 0$,

$k = -8$ or 4



Need $\boxed{-8 < k < 4}$

9. $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2} = 4x - 6x^{1/2} + 8x^{-2}$

(a) $f(x) = \int f'(x)\,dx = 4\left(\frac{x^2}{2}\right) - 6\frac{x^{3/2}}{(3/2)} + 8\left(\frac{x^{-1}}{-1}\right) + c$

$= 2x^2 - 4x^{3/2} - 8x^{-1} + c$

P is $(4,1)$ so

$f(4) = 2(4^2) - 4(4\sqrt{4}) - \frac{8}{4} + c$

$= 32 - 32 - 2 + c$

$= -2 + c = 1 \quad \therefore c = 3$

$f(x) = 2x^2 - 4x^{3/2} - 8x^{-1} + 3$ "tidied version".

(b) Gradient $= f'(4) = 16 - 6\sqrt{4} + \frac{8}{16}$

$= 16 - 12 + 1/2 = 4\,1/2 = 9/2 = m_T$

$m_N = \frac{-1}{(9/2)} = -2/9$

$y - 1 = -2/9\,(x - 4)$ ✓

$$\left.\begin{aligned} 9y - 9 &= -2(x-4) = -2x + 8 \\ 2x + 9y - 17 &= 0. \end{aligned}\right\}\ \text{not essential.}$$

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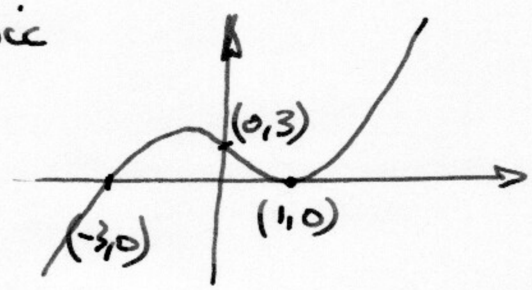
10. $y = (x+3)(x-1)^2$, roots $x = -3$, $x = 1$ (repeated)

(a) $+x^3$ cubic



(b) $y = (x+3)(x^2-2x+1) = x^3 - 2x^2 + x$
$\qquad + 3x^2 - 6x + 3$

$= x^3 + 2x^2 - 5x + 3, \quad k = 3.$

(c) $dy/dx = 3x^2 + 2x - 5 = 3$

$\therefore 3x^2 + 2x - 8 = 0 \qquad ac = -24 = 6 \times (-4)$

$\left(3x - \frac{4}{1}\right)\left(x + \frac{6}{3}\right) = 0, \quad (3x-4)(x+2) = 0$

$x = 4/3, \quad x = -2.$

11. $a = 30, \quad d = -1.5$

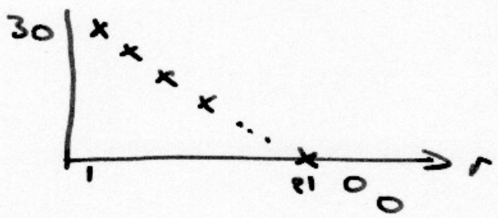
(a) $u_n = a + (n-1)d$

$u_{25} = a + 24d = 30 - 36 = -6$

(b) $u_r = 0 = 30 - 1.5(r-1)$

$\therefore r - 1 = 20, \quad r = 21$

(c) Need sum of all positive terms, S_{20} or S_{21}
(same!)



$S_{21} = \frac{21}{2}\left(2a + (n-1)d\right)$

$= \frac{21}{2}(60 \cancel{+} -30) = 21 \times 15 = 315$

or $S_{20} = \frac{20}{2}(60 - 19 \times 1.5) = 10 \times 31.5 = 315$