C1 JAN 2007

1. $y = 4x^3 - 1 + 2x^{1/2}$

$dy/dx = 4(3x^2) - 0 + 2(\tfrac{1}{2}x^{-1/2})$

$= 12x^2 + x^{-1/2}$

2. (a) $\sqrt{108} = \sqrt{3 \times 36} = 6\sqrt{3}$

Think – perfect squares 1, 4, 9, 16, 25, 36 – which of these divide into 108?

(b) $(2-\sqrt{3})^2 = (2-\sqrt{3})(2-\sqrt{3}) = 4 - 2\sqrt{3} - 2\sqrt{3} - \sqrt{3}(-\sqrt{3})$

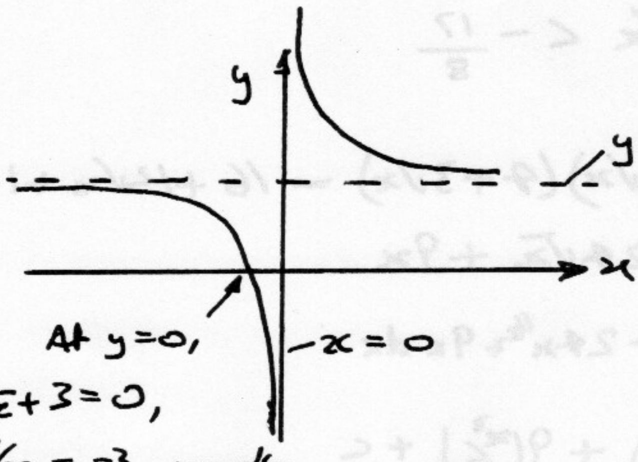
$= 4 - 4\sqrt{3} + 3$

$= 7 - 4\sqrt{3}$

Think: $(a+b)^2 = a^2 + b^2 + 2ab$, $a = 2$, $b = -\sqrt{3}$

3.

$y = f(x) + 3$ is $y = f(x)$ translated $\uparrow +3$.



At $y=0$,
$\frac{1}{x} + 3 = 0$,
$1/x = -3$, $x = -1/3$
so $\underline{(-1/3, 0)}$.

You need to know common curves: $y = 1/x$, x^2, x^3 etc

4. $y = x - 2$

$y^2 + x^2 = 10$

$\Rightarrow (x-2)^2 + x^2 = x^2 - 4x + 4 + x^2$

$= 2x^2 - 4x + 4 = 10$

$2x^2 - 4x + 4 - 10 = 2x^2 - 4x - 6 = 0$

$(\div 2)$ $x^2 - 2x - 3 = 0$

$= (x-3)(x+1)$, $x = -1$ or $+3$.

At $x = -1$, $y = x - 2 = -3$ $\qquad$ $\underline{(-1, -3) \text{ and } (3, 1)}$

At $x = 3$, $y = x - 2 = 1$

5. No real roots $\Rightarrow$ discriminant $b^2 - 4ac < 0$

$2x^2 - 3x - (k+1) =$ "$ax^2 + bx + c$" $= 0$

$b^2 - 4ac = (-3)^2 - 4 \times 2 \times -(k+1)$

$= 9 + 8(k+1) = 17 + 8k < 0$

$\therefore 8k < -17$, $k < -\frac{17}{8}$

6 (a) $(4 + 3\sqrt{x})^2 = (4 + 3\sqrt{x})(4 + 3\sqrt{x}) = 16 + 12\sqrt{x} + 12\sqrt{x} + 9x$

$= 16 + 24\sqrt{x} + 9x$

(b) $\int (4 + 3\sqrt{x})^2 dx = \int 16 + 24x^{1/2} + 9x \, dx$

$= 16x + 24\left(\frac{x^{3/2}}{3/2}\right) + 9\left(\frac{x^2}{2}\right) + c$

$= 16x + 16x^{3/2} + \frac{9}{2}x^2 + c$.

7. (a) Obviously a differential equation question - integrate, then find the constant.

$$y = f(x) = \int f'(x)\,dx = \int 3x^2 - 6 - 8x^{-2}\,dx$$

$$= 3\left(x^3/3\right) - 6x - 8\left(\frac{x^{-1}}{-1}\right) + c$$

$$= x^3 - 6x + 8x^{-1} + c.$$

At $x = 2$, $y = 2^3 - 6\times 2 + \frac{8}{2} + c$

$= 8 - 12 + 4 + c = c = 1$ (at P, (2,1)).

$\therefore\ \underline{f(x) = x^3 - 6x + 8x^{-1} + 1}$.

(b) Tangent is a straight line, $y - y_1 = m(x - x_1)$

$dy/dx = f'(x) = 3x^2 - 6 - 8/x^2$ from (a),

at $x = 2$, $dy/dx = 3\times 4 - 6 - 8/4 = 12 - 6 - 2 = 4$

Line through (2,1), gradient 4 $\Rightarrow$

$y - 1 = 4(x - 2) = 4x - 8,$

$y = 4x - 8 + 1 = 4x - 7$

8.(a) $y = 4x + 3x^{3/2} - 2x^2$

$dy/dx = 4 + 3\left(3/2\, x^{1/2}\right) - 2(2x) = 4 + \frac{9}{2}x^{1/2} - 4x$

(b) At $x = 4$, y (on C) $= 4\times 4 + 3\left(4^{3/2}\right) - 2\times 4^2$

$= 16 + 3(4\times 2) - 2\times 16 = 16 + 24 - 32$

$= 8$ so (4, 8) is on C.

(c) At $x = 4$, $dy/dx = 4 + \frac{9}{2}\sqrt{4} - 4\times 4$

$= 4 + 9 - 16 = -3$

$\therefore$ Gradient of normal $= \frac{-1}{-3} = \frac{1}{3}$

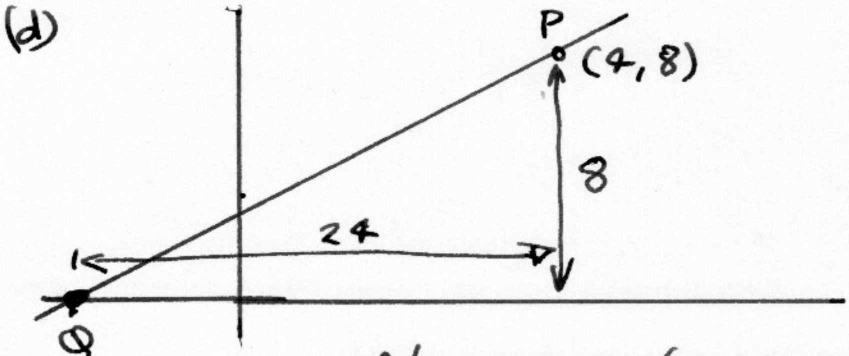
8(c) + Line through $(4, 8)$ with gradient $1/3$:

$$y - 8 = \tfrac{1}{3}(x - 4)$$

$$(\times 3) \quad 3y - 24 = x - 4$$

$$(+24) \quad \underline{3y = x + 20}$$

(d)



At $y = 0$ (on x-axis) $x + 20 = 0$,
$x = -20$.

Length $PQ = \sqrt{24^2 + 8^2} = \sqrt{576 + 64} = \sqrt{640}$

$$= \sqrt{64 \times 10} = 8\sqrt{10}$$

9(a) $n = 1, \ 2, \ 3$
$u_n = 4, \ 7, \ 10$ ($d = 3$)

$u_n = 3n + 1$

so $u_1 = 4$ etc

(b) total in 10 rows

Let $a = 4$, $d = 3$

$$S_n = \tfrac{n}{2}\left(2a + (n-1)d\right)$$

$$S_{10} = \frac{10}{2}\left(2 \times 4 + (10-1) \times 3\right) = 5(8 + 27)$$

$$= 5 \times 35 = 175$$

(c) The question implies that she can start (but not finish) row $k+1$, so

$S_k < 1750$ ($\leq$ would have none left over).

$$S_k = \frac{k}{2}\left(8 + 3(k-1)\right) = \tfrac{k}{2}(5 + 3k) < 1750,$$

(x2): $k(5+3k) < 3500$,

$3k^2+5k-3500 < 0$

Factorise, $3 \times -3500 = -10500$

Factors adding to $+5$ are 105×-100

$3k^2 + 105k - 100k - 3500$

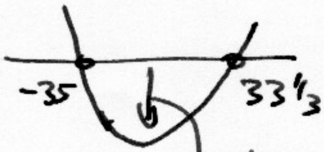
$= 3k(k+35) - 100(k+35)$

$= (3k-100)(k+35) < 0$

[or write as $\left(3k - \frac{100}{1}\right)\left(1k + \frac{105}{3}\right) = (3k-100)(k+35)$]

(d) Set $= 0$ to find critical values:

$(3k-100)(k+35) = 0$, $k = -35$ or $33\frac{1}{3}$.



need < 0 so $-35 < k < 33\frac{1}{3}$,

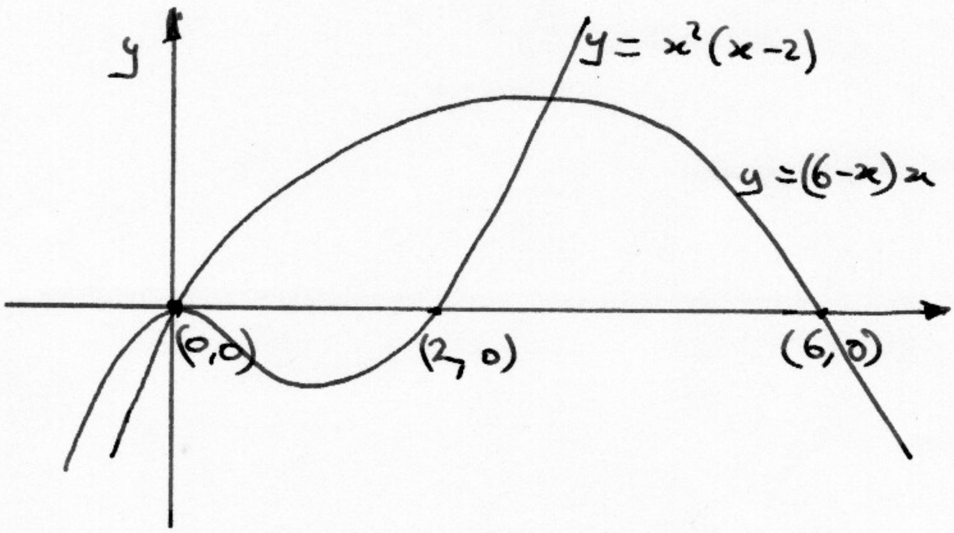
biggest integer $= \underline{\underline{33}}$.

10 (a)(i) $y = x^2(x-2)$, $y = 0$ at $x = 0$ (repeated root, just touching) and at $x = 2$.

Cubic with $a > 0$ $(ax^3 + bx^2 + cx + d)$.

so .

(ii) $y = x(6-x)$, quadratic with $-x^2$ so $\cap$, $y = 0$ at $x = 0$ and $x = 6$.



(b) when $y = x^3 - 2x^2$ and $y = 6x - x^2$ intersect, $x^3 - 2x^2 = 6x - x^2$

(both curves same x & same y at intersection point)

$\therefore \cancel{x^3 + x^2 - 8x^2 = 0}$

$x^3 - 2x^2 - 6x + x^2 = x^3 - x^2 - 6x = 0$

$= x(x^2 - x - 6)$

$\therefore x = 0$ (origin) or $x^2 - x - 6 = 0$

$= (x-3)(x+2)$, $x = 3$ or -2.

At $x = -2$, $x(6-x) = -2(6+2) = -16$

At $x = 3$, $x(6-x) = 3 \times 3 = 9$

$\therefore$ intersection points are $(-2, -16)$, $(0,0)$ and $(3,9)$.